## A Counting Exercise using Cardioids

FACT: If $\boldsymbol{n}>4$ and $\boldsymbol{n}$ is even with $\boldsymbol{k}=\frac{\boldsymbol{n}}{\boldsymbol{2}}-1$, the image consists entirely of horizontal and vertical lines.
Sometimes the image has rectangles. The number of rectangles that do not overlap can be counted using the techniques described in PART II of PwP. Call this number of rectangles $\boldsymbol{R}(\boldsymbol{n})$.

1. Examine the number of rectangles for $\boldsymbol{n}=6,8,10,12$, and 14 . Does it appear that a single formula for $\boldsymbol{R}(\boldsymbol{n})$ should work for all five of these values of $\boldsymbol{n}$ ? Briefly explain your answer and don't be afraid to say no!

NOTE: This file uses only two parameters to create the image, $\boldsymbol{n}$ and $\boldsymbol{k}$. For the next three questions, a third parameter, $\boldsymbol{j}$, is used to denote whole numbers starting at either 0 or 1 . This parameter creates $\boldsymbol{n}$ using the formula shown.
2. Determine $\boldsymbol{R}(\boldsymbol{n})$ for $\boldsymbol{n}=4 \boldsymbol{j}+6$ for $\boldsymbol{j}=0,1, \ldots$
3. Determine $\boldsymbol{R}(\boldsymbol{n})$ for $\boldsymbol{n}=8 \boldsymbol{j}+8$ for $\boldsymbol{j}=0,1, \ldots$
4. Determine $\boldsymbol{R}(\boldsymbol{n})$ for $\boldsymbol{n}=8 \boldsymbol{j}+4$ for $\boldsymbol{j}=1, \ldots$

HINTS for questions 2-4: Although this is conceptualized as $\boldsymbol{R}$ as a function of $\boldsymbol{n}$, it is easiest to consider $\boldsymbol{R}$ as a function of $\boldsymbol{j}$ given how $\boldsymbol{j}$ is related to $\boldsymbol{n}$ in each problem. From here you can rewrite $\boldsymbol{R}$ as a function of $\boldsymbol{n}$. Also note that $2+4+6$ can be reconceptualized as $2 *(1+2+3)$.

