# Calculating Lines used given Jump Sequences

Stars are created from having a vertex jump of larger than 1, the most common is the pentagram obtained from n = 5 and J = 2. The jump of 2 occurs repeatedly until the initial vertex is obtained. Jump sets work in the same fashion except two or more different jumps occur before the pattern is repeated. The image is completed once the initial vertex is achieved AT THE END OF A JUMP SEQUENCE.
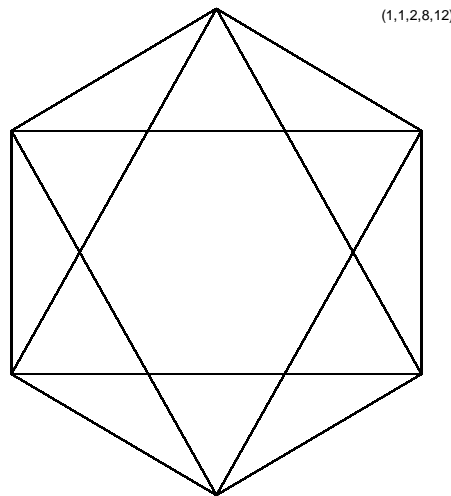
Images A-C involve S = P = 1 (so each image is the vertex frame) and n = 12 (so that it is easy to talk about jumps in "hour" terms meaning that if you are at 11 o'clock and add 3 hours you get to 2 o'clock not 14). In Panel A, J_1 = 3, J_2 = 8, leads to jumps of:

The vertex jump pattern is thus: 3,   8,   3,   8,   3,   8,   **3**,   8,   … 3,   8
In hours, the image is drawn as: 12&0 to 3 to 11 to 2 to 10 to 1 to 9 to **12&0** to 8 … to 4 to 12&0
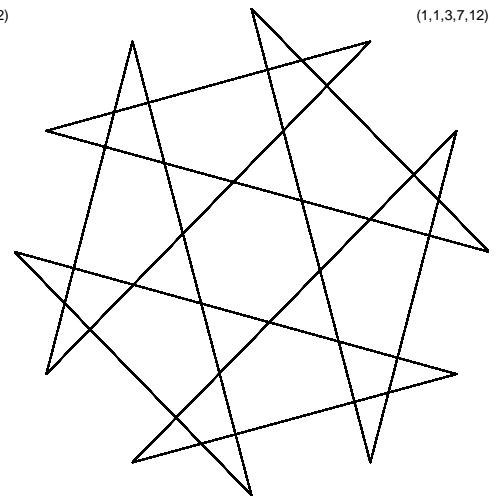
Notice, in particular, that the pattern does NOT stop at the bolded **12&0** above because the jump of **3** from 9 is not the end of the jump sequence. The end of the sequence takes 2*n = 24 jumps. Note that each vertex is used TWICE (there are 2 lines in and out of each vertex rather than 1). More generally, if there are k jumps in the sequence, the pattern of vertex usage will end after $k*n$ jumps if there is no commonality between n and the sum of the k jumps. (File 5 allows up to 3 jumps in a sequence and File 12 allows up to 9 but this file does not employ subdivisions.) In Panel A, 3+8 = 11 has no divisor in common with 12 but in Panels B and C, 2+8 = 10 and 3+7 = 10 both have 2 in common with 12 therefore half as many jumps occur before completing the vertex frame. Both are shown because they achieve this in different ways: B uses half the vertices, 6, twice each, and C uses all vertices, 12, once each.

(1,1,3,8,12)   (1,1,2,8,12)   (1,1,3,7,12)
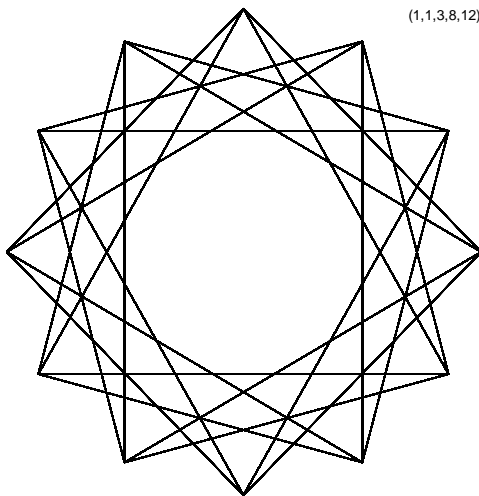
A) All vertices used twice, VCF = 1   B) Half the vertices used twice, VCF = 2   C) All vertices used once, VCF = 2
        24 lines in vertex frame   12 lines in vertex frame   12 lines in vertex frame

Calculating VCF: Given k jumps per set, let $m = J_1 + … + J_k$ be the sum of vertex jumps in one set. Then **VCF** is:

Mathematically, the *Vertex Common Factor*, **VCF**, is: **VCF = GCD(m, n)**,

where **GCD** is the greatest common divisor function.

The number of vertices used, v_{used}, (perhaps multiple times) is given by: $v_{used} = k*n/VCF$.

Calculating SCF: On each of the line segments connecting used vertices, v_{used}, we create S subdivisions leading to $S*v_{used}$ possible subdivision endpoints. Not all of these endpoints are used if P has factors in common with $S*v_{used}$.

Mathematically, the *Subdivision Common Factor*, **SCF**, is: **SCF = GCD(P, $S*v_{used}$)**.

The number of lines in the image, **L**, is then given by: $L = S*v_{used}/SCF$.

NOTE: The number of lines in the vertex frame may appear less than v_{used} because some lines are used multiple times.