## Angles from Regular Polygons and Stars



Continuously drawn stars are created from regular polygons having $n$ vertices by jumping $J$ vertices between each line. Such stars have $\boldsymbol{n}$ points if there is no common denominator between $\boldsymbol{J}$ and $\boldsymbol{n}$. All $\boldsymbol{n}>4$ except $\boldsymbol{n}=6$ has at least one $\boldsymbol{J}$ that produces an $n$-point star. Call the angle created between successive lines, the $n, J$ angle.

Imagine a star is created from a regular $\boldsymbol{n}$-gon with $\boldsymbol{J}$ jumps. In order to focus on the general rule, specific values of $\boldsymbol{n}$ and $\boldsymbol{J}$ are not provided in the image to the left. In this instance, the formula for the $\boldsymbol{n}, \boldsymbol{J}$ angle, shown in blue to the left, is:

$$
n, J \text { angle }=\frac{(n-2 * J)}{n} * 180^{\circ} \text { as long as } J<n / 2 .
$$

If $\boldsymbol{J}=1$, the image is a polygon but the same equation for determining the $\boldsymbol{n}, 1$ angle holds. This formula is provided without proof, but it is based on a rule from geometry called the Inscribed Angle Theorem. The table below applies this formula and provides angle measures for polygons and stars for $\boldsymbol{n} \leq 30$.

Angle in degrees of Regular Polygons and Stars, 3 to 30

|  | Polygon | Star jump value $\boldsymbol{J}$ ( $\boldsymbol{J}$ and $\boldsymbol{n}$ have no common factors greater than 1, and J < n/2) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{n}$ | $(J=1)$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 3 | 60 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 90 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 108 | 36 |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 120 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 | 128.57 | 77.14 | 25.71 |  |  |  |  |  |  |  |  |  |  |  |
| 8 | 135 |  | 45 |  |  |  |  |  |  |  |  |  |  |  |
| 9 | 140 | 100 |  | 20 |  |  |  |  |  |  |  |  |  |  |
| 10 | 144 |  | 72 |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 147.27 | 114.5 | 81.82 | 49.09 | 16.36 |  |  |  |  |  |  |  |  |  |
| 12 | 150 |  |  |  | 30 |  |  |  |  |  |  |  |  |  |
| 13 | 152.31 | 124.6 | 96.92 | 69.23 | 41.54 | 13.85 |  |  |  |  |  |  |  |  |
| 14 | 154.29 |  | 102.9 |  | 51.43 |  |  |  |  |  |  |  |  |  |
| 15 | 156 | 132 |  | 84 |  |  | 12 |  |  |  |  |  |  |  |
| 16 | 157.5 |  | 112.5 |  | 67.5 |  | 22.5 |  |  |  |  |  |  |  |
| 17 | 158.82 | 137.6 | 116.5 | 95.29 | 74.12 | 52.94 | 31.76 | 10.59 |  |  |  |  |  |  |
| 18 | 160 |  |  |  | 80 |  | 40 |  |  |  |  |  |  |  |
| 19 | 161.05 | 142.1 | 123.2 | 104.2 | 85.26 | 66.32 | 47.37 | 28.42 | 9.474 |  |  |  |  |  |
| 20 | 162 |  | 126 |  |  |  | 54 |  | 18 |  |  |  |  |  |
| 21 | 162.86 | 145.7 |  | 111.4 | 94.29 |  |  | 42.86 |  | 8.571 |  |  |  |  |
| 22 | 163.64 |  | 130.9 |  | 98.18 |  | 65.45 |  | 32.73 |  |  |  |  |  |
| 23 | 164.35 | 148.7 | 133 | 117.4 | 101.7 | 86.09 | 70.43 | 54.78 | 39.13 | 23.48 | 7.826 |  |  |  |
| 24 | 165 |  |  |  | 105 |  | 75 |  |  |  | 15 |  |  |  |
| 25 | 165.6 | 151.2 | 136.8 | 122.4 |  | 93.6 | 79.2 | 64.8 | 50.4 |  | 21.6 | 7.2 |  |  |
| 26 | 166.15 |  | 138.5 |  | 110.8 |  | 83.08 |  | 55.38 |  | 27.69 |  |  |  |
| 27 | 166.67 | 153.3 |  | 126.7 | 113.3 |  | 86.67 | 73.33 |  | 46.67 | 33.33 |  | 6.667 |  |
| 28 | 167.14 |  | 141.4 |  | 115.7 |  |  |  | 64.29 |  | 38.57 |  | 12.86 |  |
| 29 | 167.59 | 155.2 | 142.8 | 130.3 | 117.9 | 105.5 | 93.1 | 80.69 | 68.28 | 55.86 | 43.45 | 31.03 | 18.62 | 6.207 |
| 30 | 168 |  |  |  |  |  | 96 |  |  |  | 48 |  | 24 |  |

