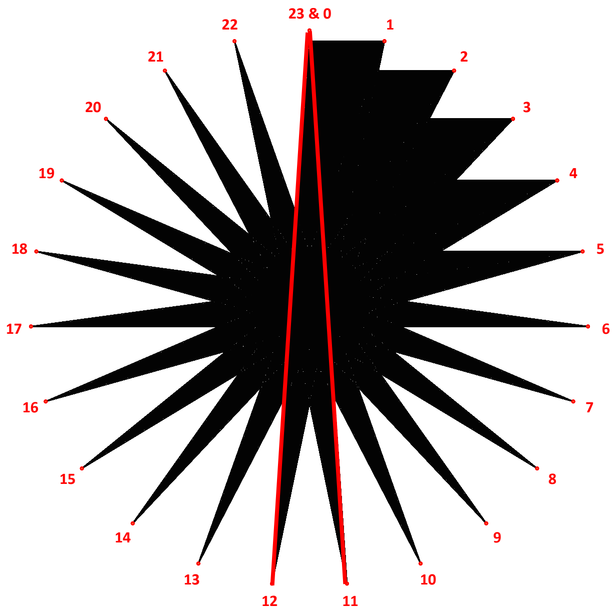
Sharpest Stars

When more than one ***n***-point star exists, some will have sharper points than others. The one with the sharpest points will have ***J*** be as close to ***n***/2 as possible without equaling ***n***/2. The reason is straightforward: If ***J*** = ***n***/2, a vertical line will result. This is only possible when ***n*** is even. A separate rule exists for even ***n*** and odd ***n***.

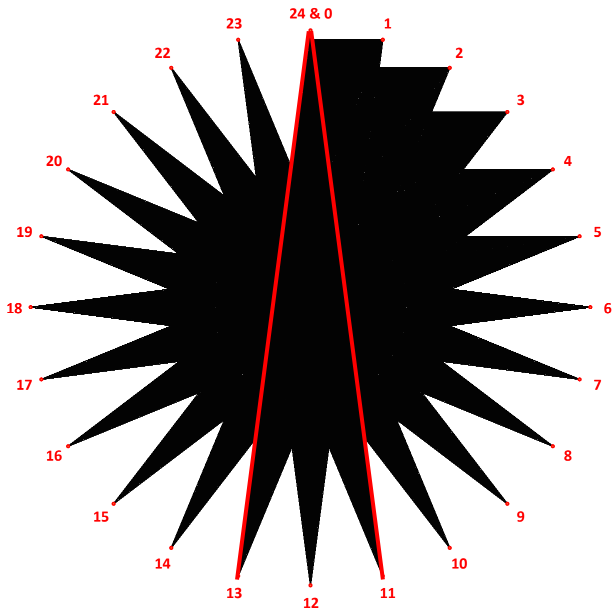
Sharpest star when *n* is even. The sharpest star occurs when ***J*** = ***n***/2 – 1.

Sharpest star when *n* is odd. The sharpest star occurs when ***J*** = (***n*** – 1)/2.

Examples of both are provided below for ***n*** = 23 and ***n*** = 24. Both have the angle at the top vertex highlighted with **red lines** over the lines forming the angle.



n = 23



n = 24

There are more vertices on the right, but the points are sharper on the left. Additionally, the hole in the center is larger on the right. This is true in general when comparing an odd ***n*** and the larger value of ***n*** (which is ***n***+1 and is even). The reason why is seen by looking at the vertices at the top and bottom of each image. These vertices create the angle at the top. The **right** side of the angle at 0 goes from 0 to 11 in both panels. This follows the **Sharpest Star Rules** given above for two values of ***n*** next to one another with the odd ***n*** being the smaller ***n***. In particular, the value of ***J*** for ***n*** = 23 is ***J*** = (23-1)/2 = 11 and the value of ***J*** for ***n*** = 24 is ***J*** = 24/2-1 = 11. The **left** side is 1 vertex over when ***n*** is odd (from 0 to 12), but two vertices over when ***n*** is even (from 0 to 13). Put another way, the sharpest odd angle spans one vertex (11 to 12) but the sharpest even angle spans two vertices (11 to 13).

Optional information for those interested in angle measurement. When ***n*** is odd, the sharpest angle is 180°/***n*** and when ***n*** is even, the sharpest angle is 360°/***n***. (This follows from a rule of geometry called the *Inscribed Angle Theorem*.) This angle is the same for every vertex point in a regular star. The two images above thus have angles of 180°/23 = 7.83° on the left, and 360°/24 = 15° on the right. You can verify that these equations work by looking at ***n*** = 3 which produces an equilateral triangle (with 60° angles) and ***n*** = 4 which produces a square (with 90° angles).