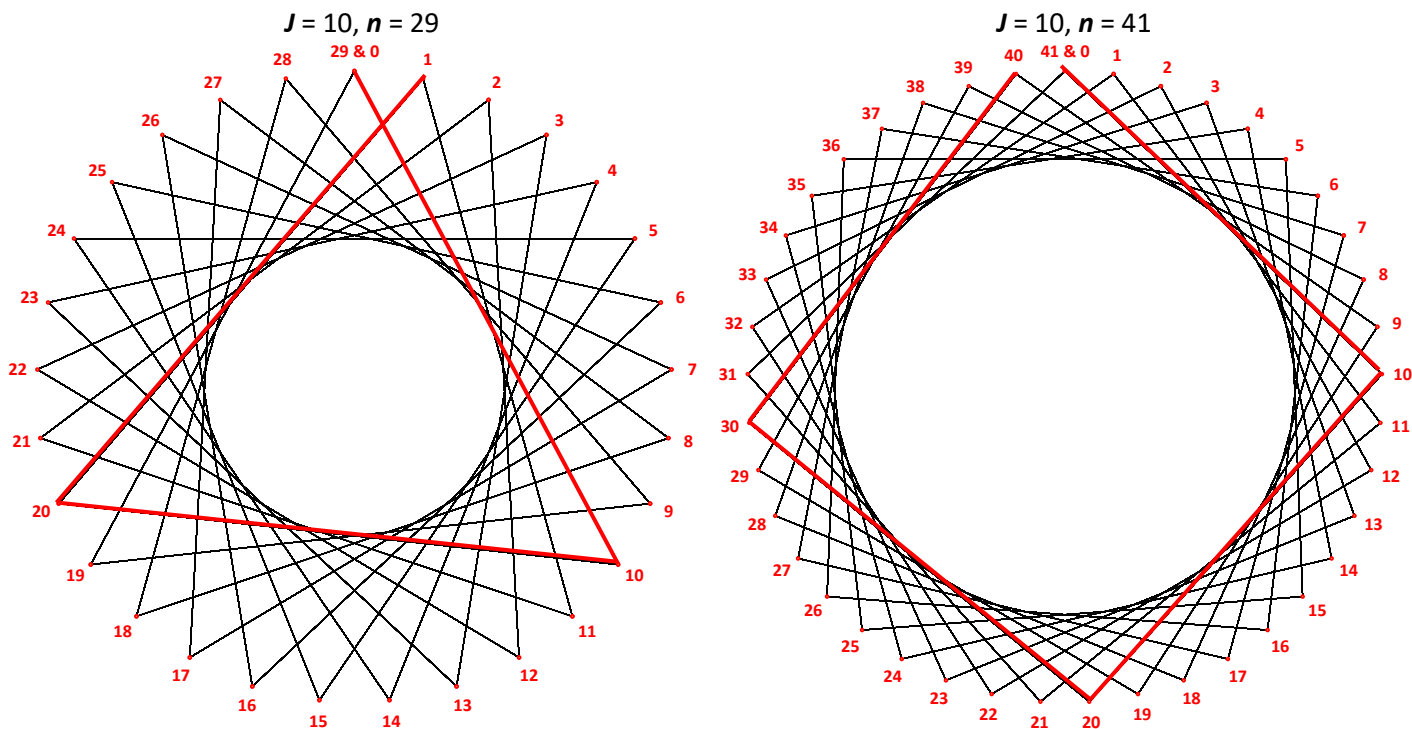


Viewing Stars as Rotating Polygons

When n is a multiple of J then the image collapses to that common multiple. For example, if $J = 10$ and $n = 30$ a triangle results but if $n = 40$ a square results. On the other hand, if n is close to 30 or close to 40, then the star that results can be thought of as being constructed by rotating triangles or squares. (These images are NOT triangles or squares but are close to those images.) The following are two examples which have been annotated with the first 3 and 4 segments.



“One-off” images. The left image is a bit more than once around (since $30 > n$) and the right image is not quite once around (since $40 < n$). The next iteration will move to 2 on the left and 39 on the right. This will continue until the last iteration concludes with a segment from 19 to 29 on the left and from 31 to 41 on the right. The triangles on the left rotate clockwise, while the squares on the right rotate counterclockwise. If you want to see these rotations reverse, just change to $n = 31$ for triangles and $n = 39$ for squares. And if you want to see a rotating pentagon, set $n = 49$ or 51.

This link takes you to the first image. Click *Toggle Drawing* to watch the image emerge in $1/3$ clockwise turn. Then change n to 29 to see the reverse. Do the same for the image at the right by changing n to 39 and 41 in which case the image emerges in a quarter turn clockwise or counterclockwise.

<https://www.playingwithpolygons.com?vertex=29&subdivisions=1&points=1&jumps=10>

“Multiple-off” images. If you try 2-off versions of n (28 and 32 or 38 and 42) then every other vertex is skipped due to the common factor (of 2) with these values of n with $J = 10$. If we choose J to be a prime number, we no longer have this issue. Additionally, if we choose larger values of J and n , the rotating images are easier to see. As a result, consider the following starting point based on $J = 67$ and $n = 200$ (which produces a rotating triangle). Click *Toggle Drawing*.

<https://www.playingwithpolygons.com?vertex=200&subdivisions=1&points=1&jumps=67>

$n = 200$ is just under $n = 3*J$ ($3*67 = 201$) so this rotates clockwise just like the left image above. Change n to 202 and the rotation reverses. Both are “one-off” images. As a result, the image is completed after $1/3$ rotation of the triangle.

Try instead the 2-off image, $n = 199$ or $n = 203$ ($n = 3*J \pm 2$). What happens now after $1/3$ rotation? How much rotation is required to complete the image? What happens if $n = 198$ or $n = 204$? What happens after $1/3$ rotation? What happens after $2/3$ rotation? How many thirds does it take to complete the image in this instance? What happens as n gets further away from $3*J$? Can you come up with a rule to tell how many rotations it takes to complete the image?

Other pairs to consider are $n = 267$ and 269, or 334 and 336. One final pair to consider are $n = 167$ and 168. A rotating pentagon occurs if n is near $n = 2.5*J$ (2.5 makes sense when you note that a pentagon is 5 jump 2).