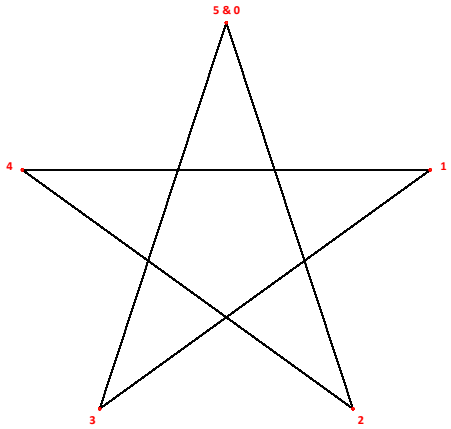
Stars that Work and Stars that Don't

A **continuously drawn star** is created by connecting vertices that are a defined number of jumps, ***J***, away from one another until the image is complete. When ***J*** = 1, a polygons results. If ***J*** > 1, stars can, but do not have to, result. Such stars are completed in ***n*** segments.



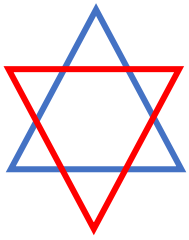
The simplest example is the 5-point star (or pentagram) which occurs when every other vertex is connected, ***n*** = 5 and ***J*** = 2.

The jumps involved are (→ means jump): 0→2→4→1→3→5=0, and the image is complete after 2 clockwise trips around the circle (jumping from larger to smaller number means going around again).

The same image results if ***J*** = 3, with jumps of 0→3→1→4→2→5=0. The only difference in this instance is that it takes 3 clockwise trips around the circle to complete the circuit.

On *J* < *n*/2. The same image occurs for jumps of ***J*** and ***n*-*J***, the only difference being the number of times around the circle required to complete the circuit. For this reason, we typically restrict our discussion to 1 < ***J*** < ***n***/2.

When stars don't exist. If ***J*** and ***n*** have a factor in common, the circuit is completed in less than ***n*** jumps. The simplest example of this is ***n*** = 6 and the only ***J*** between 1 and 3 = ***n***/2 is 2. But 2 has a factor (2) in common with 6, therefore no continuously drawn 6-point star exists. The only way to obtain a 6-point star is to create a second circuit such as the blue and red circuits to the right.



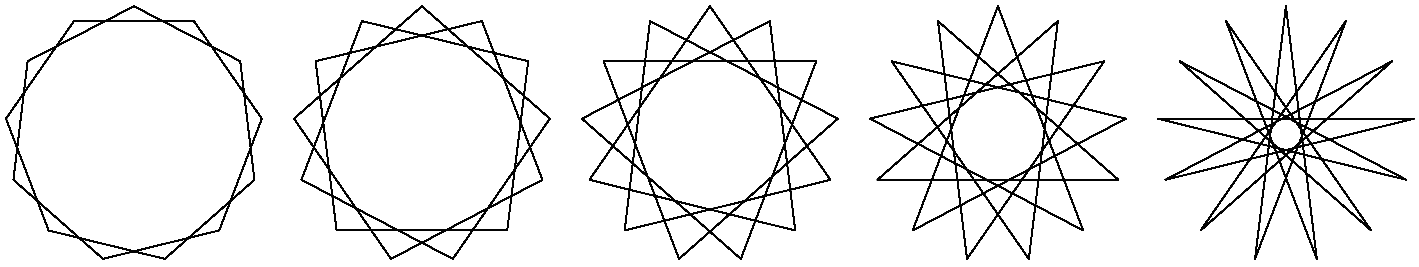
We only deal with continuously drawn stars in this book.

How many stars are there? An ***n***-point star exists whenever ***n*** and ***J*** have no factors in common. This means that there are a greater number of different ***n***-point stars if ***n*** is prime than surrounding values of ***n***.

For example, consider values of ***n*** close to the prime number 13. How many different continuously drawn stars exist for ***n*** from 12 ≤ ***n*** ≤ 16? The answers are:

1 for ***n*** = 12; 5 for ***n*** = 13; 2 for ***n*** = 14; 3 for ***n*** = 15; 3 for ***n*** = 16.

You should use the file to verify that this is the case. The five stars for ***n*** = 13 are shown below (with 2 ≤ ***J*** ≤ 6).



Challenge question: Without using the file, answer the following: How many different 17-point stars exist? Use the file to verify your answer.