## Variations on Sunbursts: Increasing $S$ beyond 2 while maintaining polygonal status

All the sunbursts previously examined were polygons with $\boldsymbol{S}=2$. We can still obtain interesting alternative polygons when we increase $\boldsymbol{S}$ beyond 2 . The strategy for finding $\boldsymbol{P}$ in $\boldsymbol{S}=2$ sunbursts applies here: you want the "nearest" subdivision to be the first and last point used to create the image. A relation that often works is to set $\boldsymbol{J}$ and $\boldsymbol{P}$ relative to the number of lines used to create the image, $\boldsymbol{S}^{*} \boldsymbol{n}$. In particular, choose $\boldsymbol{J}$ and $\boldsymbol{P}$ such that $\boldsymbol{S}^{*} \boldsymbol{n} \pm 1=\boldsymbol{J} \boldsymbol{P}$.

Not all values of $\boldsymbol{J}$ and $\boldsymbol{P}$ satisfying this equation produce polygons. If $\boldsymbol{J}$ is "too large" the lines cross and the image is no longer a polygon. These three $\boldsymbol{n}=17, \boldsymbol{S}=3$ examples show that $\boldsymbol{J}=2$ and 4 work but 5 does not, despite having $\boldsymbol{J} * \boldsymbol{P}=50$.


51 lines $(\mathrm{S}, \mathrm{P}, \mathrm{J}, \mathrm{n})=(3,13,4,17)$


51 lines $(S, P, J, n)=(3,10,5,17)$


Note that when $\boldsymbol{S}=3$, flats occur between spikes. Sometimes the spikes can be substantial as the $\boldsymbol{n}=13, \boldsymbol{J}=4$ version at this link shows: https://www.playingwithpolygons.com?vertex=13\&subdivisions=3\&points=10\&jumps=4

For larger $\boldsymbol{S}$, note that if $\boldsymbol{S}$ is even, then $\boldsymbol{J}$ must be odd in order to not have a subdivision point right beneath a vertex. Below are two examples, one for $S=4$, the other for $S=5$.
$(S, P, J, n)=(4,15,3,11)$
44 lines $(S, P, J, \mathrm{n})=(5,28,7,39)$
195 lines



The link below takes you to an $\boldsymbol{n}=23, \boldsymbol{S}=7$ example where $\boldsymbol{J}=5$. As it turns out, for this $\boldsymbol{n}, \boldsymbol{S}$ combination, the largest $\boldsymbol{J}$ for which the image is still a polygon is $\boldsymbol{J}=5$. Note that when $\boldsymbol{J}=6$ and $\boldsymbol{P}=27$, crossovers occur. As an exercise, find $\boldsymbol{J}$ and $\boldsymbol{P}$ combinations whose product is either 160 or 162 for $\boldsymbol{J}=2,3,4$.

