## Description of how Lines are Created from Subdivisions (without multiplication)

The most important thing to understand in PART I is the subdivision counting rule. Subdivisions are equally spaced segments between vertices. If $S$ is 2 then there are two subdivisions between the starting point and the first vertex, and two more between first and second, and so on. Each of the intermediate points would be a midpoint between vertices so that there are twice as many points to use if we include the midpoints along with vertices of the polygon as line segment endpoints. So, if $\boldsymbol{n}=4$ and $\boldsymbol{S}=2$ we would have 8 points that can be used as the endpoint of a line segment.

Suppose we want to connect every third subdivision endpoint with a line segment. Call this $P=3$ where $P$ stands for the number of subdivisions between points. Subdivisions are counted clockwise starting at the top, then draw a line every third subdivision. There are multiple ways to count off subdivisions to create line segments but the simplest is to simply say: 1, 2, 3 draw line; 1, 2, 3 draw line; $1,2,3$ draw line; ... . Once this is done 8 times, you end up back at the top and you have created an 8-point star using a square frame. A version of what that looks like is provided below with annotations in color denoting counting subdivisions around the circle (on which the square is inscribed) with PINK first time around, RED second, and DARK RED third. Line segments are numbered in BLUE. Follow from 0 to 1 to ... to 8 to see the image created as 8 connected segments:

showing this image. To see image
created, click Toggle Drawing: https://www.playingwithpolygons.com?vertex=4\&subdivisions=2\&points=3

Here are two more examples, both use triangles with 3 subdivisions ( $n=3, s=3$ ), so there are 9 possible endpoints for line segments. The image connects 0 to 1 ... to 9 .


This is $P=2$, Hexagon in Triangle


This is $P=4$, 9-point Triangular Star
The only other image that can occur when $\boldsymbol{n}=\mathbf{3}$ and $S=\mathbf{3}$ is a triangle. You can use the file to see that a triangle occurs when $P=1,3,6$, and 8 and the above images occur twice. Images typically occur twice for $P<n^{*} S$ because $P$ and $\boldsymbol{n}^{*} S-P$ produce the same image (but require going around the triangle a different number of times). The table below shows the possibilities.

| Given $\mathrm{n}=3, \mathrm{~S}=3$, and $\mathrm{P}=$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Image: Triangle, $\triangle \square_{\text {-in- } \triangle}$ |  |  | $\triangle$ | 9-pt. $\triangle$-lar $\begin{gathered} \\ 3\end{gathered}$ | 9-pt. $\triangle$ - $\operatorname{lar} 3$ | $\triangle$ | $\square$-in- $\triangle$ | $\triangle$ |
| Segments required: | 9 | 9 | 3 | 9 | 9 | 3 | 9 | 9 |
| Times around triangle: | 1 | 2 | 1 | 4 | 5 | 2 | 7 | 8 |

