The Role of $J$ in Quivering Polygonal Images


The image above has been annotated to help explain why there are $\boldsymbol{J}$ quivering polygons in each image. Here, $\boldsymbol{J}=10$. This image is based on $\boldsymbol{n}=\boldsymbol{P}=\boldsymbol{s}$-1 so that the apparent square peaks quiver in a clockwise direction. To reduce clutter, only the odd peaks have been annotated here (with labels $1,3,5,7$ and 9 ), located on the line segments emanating from those five vertices of the 39 -gon. Each of these line segments is part of the vertex frame.

On the number of peaks: As noted in Quivering Polygons, the $1^{\text {st }}$ peak (labelled 1 ) is $9 / 10$ of the way from 30 to 1 (on the blue line) or to note the other side of the coin, it is $1 / 10$ of the way from vertex 1 on this line. Similarly, the second peak is $2 / 10$ on the way from the (not shown) line from 2 to 31 ). And the third (labelled 3 ) is $3 / 10$ away from 3 on the blue line from 3 to 32 . The same is true for all other vertices as well.

Half of the peaks have been labelled above and we have discussed why the locations are where they are. The other half (here five) are located in pairs ( 2 and 8,4 and 6 ) at the same height as one another, plus the peak of the parent polygon (at $39 \& 0$ ). Note that the only parent vertex that is part of quivering images is this peak (at $39 \& 0$ or at $41 \& 0$ below).

Note that if $\boldsymbol{J}$ is even, there is a SINGLE lowest peak that occurs on the line connecting $\boldsymbol{J} / 2 \mathrm{with} \boldsymbol{n}-\boldsymbol{J} / 2$. This line is horizontal. Further, the peak is located as the MIDPOINT of that line $((J / 2) / J$ of the way from $J / 2$ to $n-J / 2)$. When $n=39$, that occurs on the green line from 5 to 34 . If $J$ is odd, the two lowest peaks occur at lines $(J-1) / 2$ to $\boldsymbol{n}-(j+1) / 2$ and $(J+1) / 2$ to $n-(j-1) / 2$. The first of these will be upward sloping and the second will be downward sloping. More generally,

When the line emanates from any vertex $0<\boldsymbol{v}<\boldsymbol{J} / 2$ that line will be positively sloped (because $\boldsymbol{n}-\boldsymbol{J}+\boldsymbol{v}$ is below $\boldsymbol{v}$ ). Two such lines are provided in the annotated image, both are blue.

When the line emanates from any vertex $\boldsymbol{J} / 2<\boldsymbol{v}<\boldsymbol{J}$ that line will be negatively sloped (because $\boldsymbol{n}-\boldsymbol{J}+\boldsymbol{v}$ is above $\boldsymbol{v}$ ). Two such lines are provided in the annotated image, both are red.

On the direction of rotation: Consider the direction of peak rotation of the quivering polygons. With $\boldsymbol{n}=\boldsymbol{P}=\boldsymbol{S}-1$, the peaks quiver in a clockwise direction. The initial peaks (aside from the starting point) are in Quadrant I (1, 2, 3, and 4 here). When $J / 2^{\text {nd }}$ peak (when $J$ is even) is on the border between Quadrants I and II. And the rest of the peaks ( $6,7,8$, and 9 here) are in Quadrant II.

Compare this with $\boldsymbol{n}=\boldsymbol{P}=41=\boldsymbol{S}+1$ shown below. The first peak (labelled) is $4 / 40^{\text {th }}=1 / 10$ of the way from vertex 40 to vertex 9 (since 4 segments means 84 subdivisions given $\boldsymbol{P}=41$. This point is on the negatively sloped $5^{\text {th }}$ line of the vertex frame (since $\boldsymbol{S}=40$ ) and is in Quadrant II. The same will be true for vertices 2,3 and 4 . The $5^{\text {th }}$ vertex is once again halfway between vertices 36 and 5 and the last half are on positively sloped lines vertex frame lines with endpoints 35 and 4,34 and 3,33 and 2,32 and 1 , and 31 and 0 . Each of these peaks is in Quadrant $I$. In this instance, the peaks in this image are sequentially created in a counterclockwise fashion.

https://www.playingwithpolygons.com?vertex=39\&subdivisions=40\&points=39\&jumps=10

