

Stacked Circles, Take 3: Variations on the Theme

Stacked circles, as initially conceptualized, required four things:

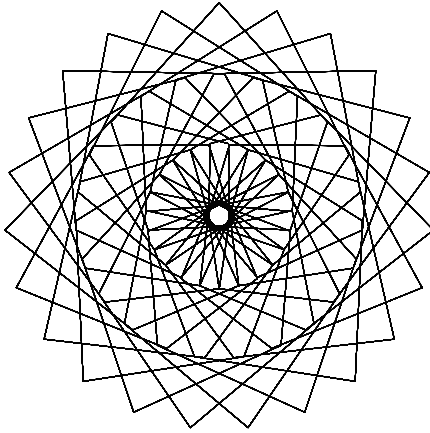
- 1) odd n ,
- 2) $J = (n-1)/2$,
- 3) even S ,
- and
- 4) P is just-under an even multiple of S , $P = 2kS - 1$.

The resulting images have cycles that loop around the center of the circle. This creates an image with a tight-closed circle for the inner-most layer and cross-hatching that does not extend into neighboring layers.

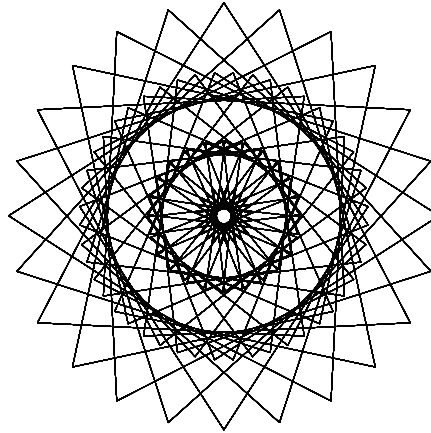
- a) What happens when we relax each of these conditions without changing the other conditions ?
- b) Can we find alternative conditions that produce versions of stacked circles?

Question a) We start from the 3 layer image annotated in *Stacked Circles, Take 1*, $(S, P, J, n) = (6, 35, 11, 23)$, shown to the left, and change each condition.

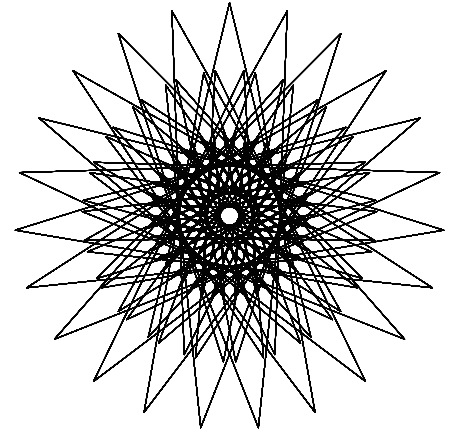
$(S, P, J, n) = (6, 35, 11, 23)$



138 lines $(S, P, J, n) = (6, 35, 11, 24)$



144 lines $(S, P, J, n) = (6, 35, 10, 23)$



138 lines

1a) Changing to even n produces significant overlap between layers even though the images are *One Layer Change* images. For example, changing $n = 23$ to 24 (middle image above) produces two levels of cross-hatching as well as a circle that is noticeably inside the subdivision vertices creating each layer (click this url and click *Subdivisions* to see):

<https://www.playingwithpolygons.com?vertex=24&subdivisions=6&points=35&jumps=11>

2a) Changing J to a value further away from the center (from 11 to 10 given $n = 23$, right image above) produces even greater amounts of apparent movement across levels (note that the image in this instance is more star-like).

<https://www.playingwithpolygons.com?vertex=23&subdivisions=6&points=35&jumps=10>

3a) Changing S to the next larger or smaller number (7 or 5) reduces the image to a simple star because the value of P (35) is a multiple of both. But, to be fair, P was defined as a function of S . If we maintain the relationship, $P = 6S - 1$, then the nearest odd choices are $S = 5$, $P = 29$ or $S = 7$, $P = 41$, both of which produce images that could be mistaken as stacked circles. Both versions are provided here:

<https://www.playingwithpolygons.com?vertex=23&subdivisions=5&points=29&jumps=11>

<https://www.playingwithpolygons.com?vertex=23&subdivisions=7&points=41&jumps=11>

Careful viewing of *Toggle Drawing* shows that in both instances, the loops no longer include the center.

4a) Changing P to the just-over even multiple of S , $P = 2kS + 1$, produces images that are also quite similar to stacked circles. But note that, as with 3a), the loops no longer contain the center.

<https://www.playingwithpolygons.com?vertex=23&subdivisions=6&points=37&jumps=11>

Question b) We see that odd S and just-over P both produce alternative versions of stacked circles. It remains to be seen if we can find a version of stacked circles for even n and for $J < (n-1)/2$.

1b) If we combine an even n with a just-over odd multiple of S (here is $P = 6*9+1$) we obtain a credible version of stacked circles. The loop includes the center even though P is a just-over value. Just-under does not work quite as well here (try $P = 53$ and note that the center is no longer included).

<https://www.playingwithpolygons.com?vertex=24&subdivisions=6&points=55&jumps=11>

2b) Changing to $J = 10$ requires a larger P to create the loop required to make the stacked circle image. The best version here is $P = 53$ which, as just-noted, is just-under $9S$. The 6-segment loop in this instance only rotates 1 vertex over for each time (click *Toggle Drawing* to see this progression).

<https://www.playingwithpolygons.com?vertex=23&subdivisions=6&points=53&jumps=10>

Notice that the loops now include the center (unlike in **1b**) with P just-below.

We see that the class of images that might be considered stacked circles is wider than initially conceptualized. All images are *One Level Change* images, but the 4 restrictions initially imposed need not apply.