## Subdivisions Create Concentric Circles of Possible Points

One of the things that happens when subdivisions are created between vertices is that these subdivisions create concentric circles of interior points. If $\boldsymbol{S}>\boldsymbol{1}$, then $\boldsymbol{S}$ is either even or odd and therefore can be written as either $\boldsymbol{S}=\mathbf{2 k}$ or $\boldsymbol{S}=\mathbf{2 k}+\mathbf{1}$. This formal way of talking about even and odd is done to highlight $\boldsymbol{k}$, the number of concentric circles and levels from Level 1 to Level $\boldsymbol{k}$. Consider $\boldsymbol{k}=1$ (and $\boldsymbol{S}=2$ or 3 ). There is a single internal circle in this instance. But $\boldsymbol{S}=4$ or 5 produces $\boldsymbol{k}=2$ levels of concentric circles. The concentric circles come from the symmetry about the midpoint of any vertex frame line. Points that are the same number of subdivisions from an end are equidistant from the circle's center.

Take $\boldsymbol{S}=4$. The $1^{\text {st }}$ (subdivision) point and the $3^{\text {rd }}$ point $(3=4-1)$ are at the same distance from the center of the circle on Level 1 and the $2^{\text {nd }}$ point (which is also the midpoint) is closer to the center than points 1 and 3.

Take $\boldsymbol{S}=5$. The $1^{\text {st }}$ (subdivision) point and the $4^{\text {th }}$ point ( $4=5-1$ ) are at the same distance from the center of the circle on Level 1 and the $2^{\text {nd }}$ and $3^{\text {rd }}$ point $(3=5-2)$ are at the same distance from the center of the circle but are closer to the center than points 1 and 4 on Level 2 .

These concentric circles are easiest to see if there are a large number of vertices (large $\boldsymbol{n}$ ) and if the number of jumps between vertices is larger than one or two. That way you see more interior points, and those points are more separated from one another rather than being packed tightly together near the outer edge of the image (since the midpoints become closer to the center). In order to distinguish between points on different interior circles, one should not choose $J$ too close to the center. The two images below show this point. Each image highlights the first line of the vertex frame.

Both images below have $\boldsymbol{S}=\boldsymbol{P}$ so that both images show the same vertex frame since $\boldsymbol{n}=19$ and $\boldsymbol{J}=8$ for both images. And both have four concentric interior circles ( $\boldsymbol{k}=4$ when $\boldsymbol{S}=8$ or 9 ). However, it is worth noting that those circles are not the same for two reasons:

1) Even $\boldsymbol{S}$ : When $\boldsymbol{S}$ is even, $\boldsymbol{S}=\mathbf{2 k}$, all but the smallest interior circle has $2 \boldsymbol{n}$ points ( 38 here). The smallest interior circle, Level 4 , has only $\boldsymbol{n}$ points ( 19 here). In this instance, $\boldsymbol{k}$ is the (single) midpoint on the smallest circle ( 4 here).

Odd $\boldsymbol{S}$ : When $\boldsymbol{S}$ is odd, $\boldsymbol{S}=\mathbf{2 k}+1$, and all interior circles have $2 \boldsymbol{n}$ points ( 38 here). In this instance, the smallest interior circle, Level 4 , has subdivision points $\boldsymbol{k}$ and $\boldsymbol{k}+1$ equidistant from the center.
2) The interior circles are a bit larger when $\boldsymbol{S}=9$ than when $\boldsymbol{S}=8$.


