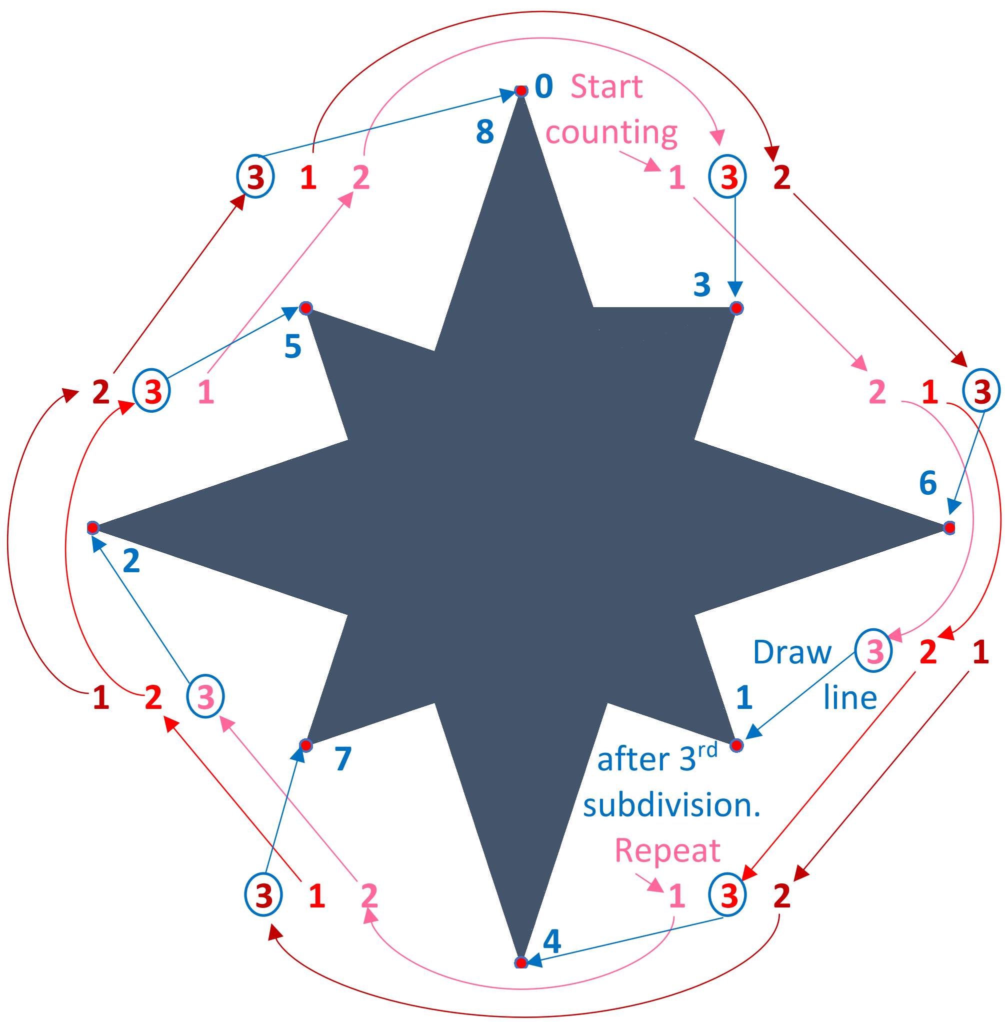
From: https://blogs.dickinson.edu/playing-with-polygons/ File 2

# Description of how Lines are Created from Subdivisions (with multiplication)

The most important thing to understand in **PART I** is the subdivision counting rule. Subdivisions are equally spaced segments between vertices. If $S$ is **2** then there are two subdivisions between the starting point and the first vertex, and two more between first and second, and so on. Each of the intermediate points would be a midpoint between vertices so that there are twice as many points to use if we include the midpoints along with vertices of the polygon as line segment endpoints. So, if $n = 4$ and $S = 2$ we would have 8 points that can be used as the endpoint of a line segment.

Suppose we want to connect every third subdivision endpoint with a line segment. Call this $P = 3$ where $P$ stands for the number of subdivisions between points. Subdivisions are counted clockwise starting at the top, then draw a line every third subdivision. There are multiple ways to count off subdivisions to create line segments but the simplest is to simply say: **1**, **2**, **3 draw line**; **1**, **2**, **3 draw line**; **1**, **2**, **3 draw line**; … . Once this is done 8 times, you end up back at the top and you have created an 8-point star using a square frame. A version of what that looks like is provided below with annotations in color denoting counting subdivisions around the circle (on which the square is inscribed) with **PINK** first time around, **RED** second, and **DARK RED** third. Line segments are numbered in **BLUE**. Follow from **0** to **1** to … to **8** to see the image created as 8 connected segments:




The location of the $j^{th}$ endpoint is easily obtained using the remainder function (MOD). Given $n*S$ possible endpoints, the $j^{th}$ is located at $r = \text{MOD}(j*P, n*S)$, so the 7th is at 5 = MOD(7*3, 4*2) above. This is the midpoint between the 2nd and 3rd vertex because $r/S = 5/2 = 2.5$ in this instance and it was attained during the third time around the circle since $2 < (j*P)/(n*S) = 21/8 < 3$.

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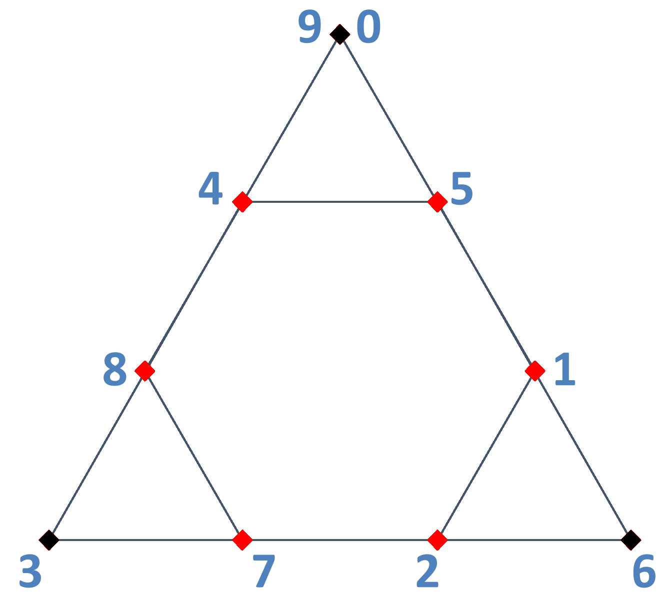
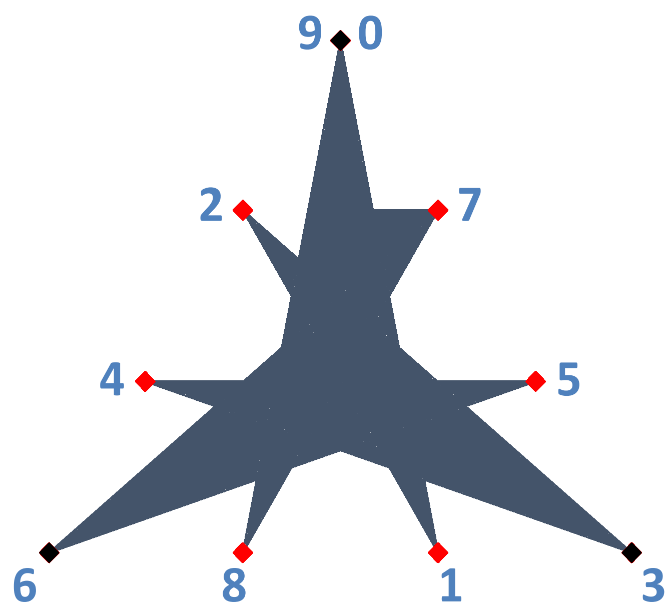
Use this link to the companion website and click *Toggle Drawing* to watch the 8 point star connect from segment to segment: https://www.playingwithpolygons.com?vertex=4&subdivisions=2&points=3

**Here are two more examples, both use triangles with 3 subdivisions (*n* = 3, *S* = 3), so there are 9 possible endpoints for line segments. The image connects numbers 0 to 1 … to 9.**




**This is *P* = 2, Hexagon in Triangle**




**This is *P* = 4, 9-point Triangular Star**

The only other image that can occur when ***n* = 3** and ***S* = 3** is a triangle. You can use the file to see that a triangle occurs when ***P*** = 1, 3, 6, and 8 and the above images occur twice. Images typically occur twice for ***P*** < ***n****S*** because ***P*** and ***n****S*** – ***P*** produce the same image (but require going around the triangle a different number of times). The table below shows the possibilities.

| **Given n = 3, S = 3**, and P = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Image: | Triangle, △ | ⬡-in-△ | △ | 9-pt.△-lar ☆ | 9-pt.△-lar ☆ | △ | ⬡-in-△ | △ |
| Segments required: | 9 | 9 | 3 | 9 | 9 | 3 | 9 | 9 |
| Times around triangle: | 1 | 2 | 1 | 4 | 5 | 2 | 7 | 8 |