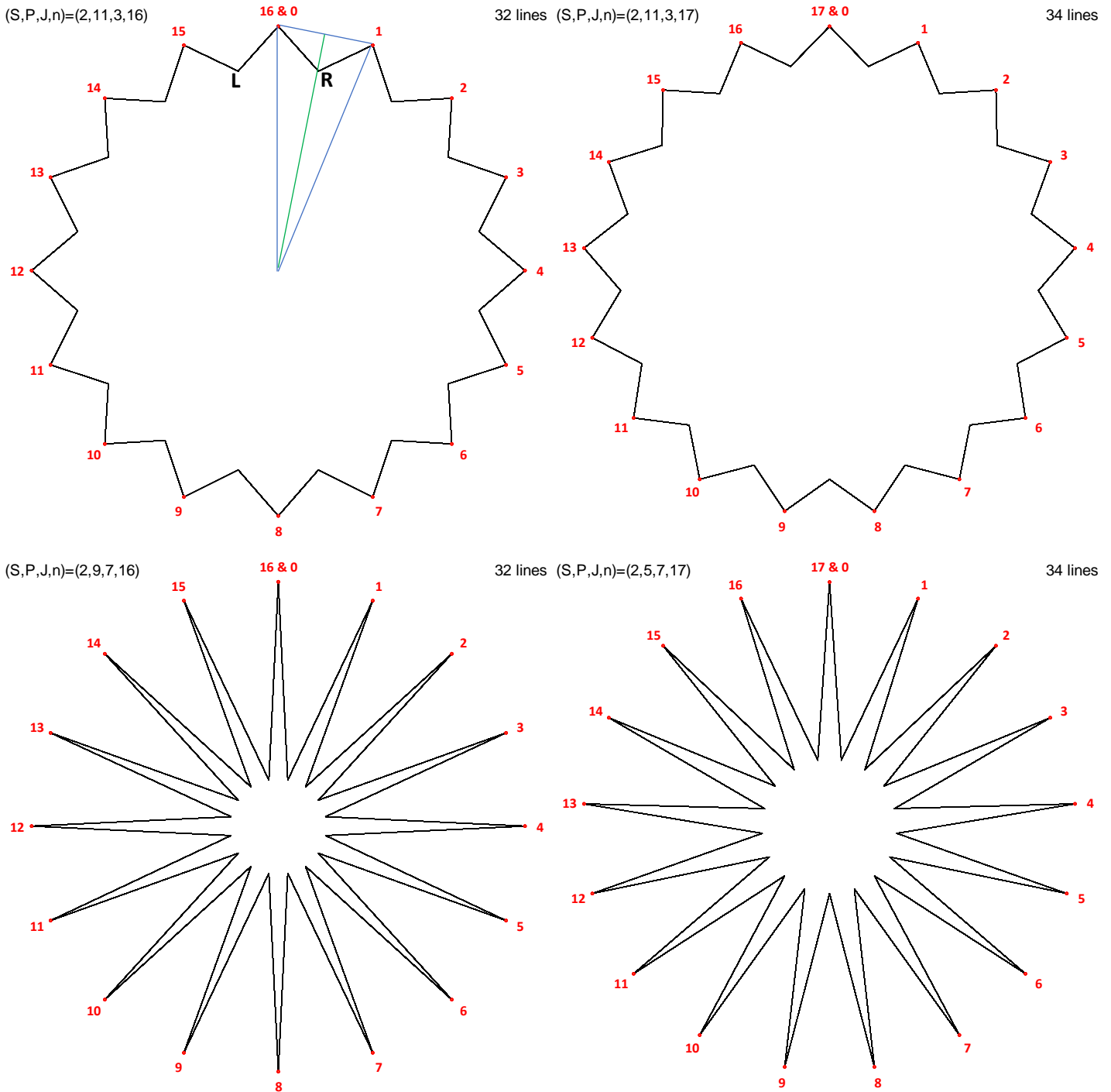


# Sunbursts

These images look like stars except that they do not have crossing lines. Since the lines do not cross, they are formally polygons. Sunbursts can have an even or odd number of spikes and the spikes can be either small or large as the images below confirm. The two left images have  $n = 16$  while the right have  $n = 17$ .



**Making sunbursts.** Sunbursts are easy to create. Imagine the polygon as being made of  $n$  equal-sized pieces cut in straight lines to the center. Each slice is an isosceles triangle (the upper left image shows one such slice in blue). The green line from the peak (center) to the midpoint of the base (from 0 to 1) is the height of the triangle. The inner point of the sunburst (like R) must be on this green line. Additionally, the first such inner point must be on one of two centerlines, the one annotated above from 0 to 1 (at R, for example) or the one from  $n-1$  to  $n$  (at L, just to the left of the centerline) since we do not want image segments to cross one another. These requirements create three conditions.

1. We guarantee midpoints if  $S = 2$ .
2.  $J$  is odd,  $J = 2k+1$  for whole number  $k$ . (If  $J$  is even, midpoints are on rays through vertices not between vertices.)
3.  $P$  is set so that the endpoint of the first segment is the midpoint of either the line from  $n-k-1$  to  $k$  or  $n-k$  to  $k+1$ .

As  $J$  increases for a given  $n$ , the spikes increase in size. The four examples below are based on  $n = 19$ , the smallest  $n$  that will produce 4 distinct sunbursts. Note also that  $\text{GCD}(n, J) = 1$ ;  $n = 15, J = 3$  will not produce a sunburst with 15 spikes.

**An example.** Take the upper right image below (where  $J = 5$  so  $k = 2$ ). The midpoint of the part of the vertex frame from  $n-k-1$  to  $k$  is the midpoint of  $19-2-1 = 16$  to  $2$  which is the point between  $18$  and  $19$ . The midpoint of  $n-k$  to  $k+1$  is the midpoint of  $19-2 = 17$  to  $3$  which is the point between  $0$  and  $1$ . We choose to use the smallest  $P$  that works (remember,  $P$  and  $n \cdot S - P$  produce the same image). Each jump of the vertex frame is  $5$  the progression goes  $0$  to  $5$  to  $10$  to  $15$  to  $1$  to  $6$  to  $11$  to  $16$ . Therefore,  $16$  is the first (of  $16$  and  $17$ ) and this occurs after  $7$  jumps, so  $P = 2 \cdot 7 + 1 = 15$ . This is drawn counterclockwise. <https://www.playingwithpolygons.com?vertex=19&subdivisions=2&points=15&jumps=5>

