A Counting Triangles Exercise Using Dan Meyer's *Three Act Math*

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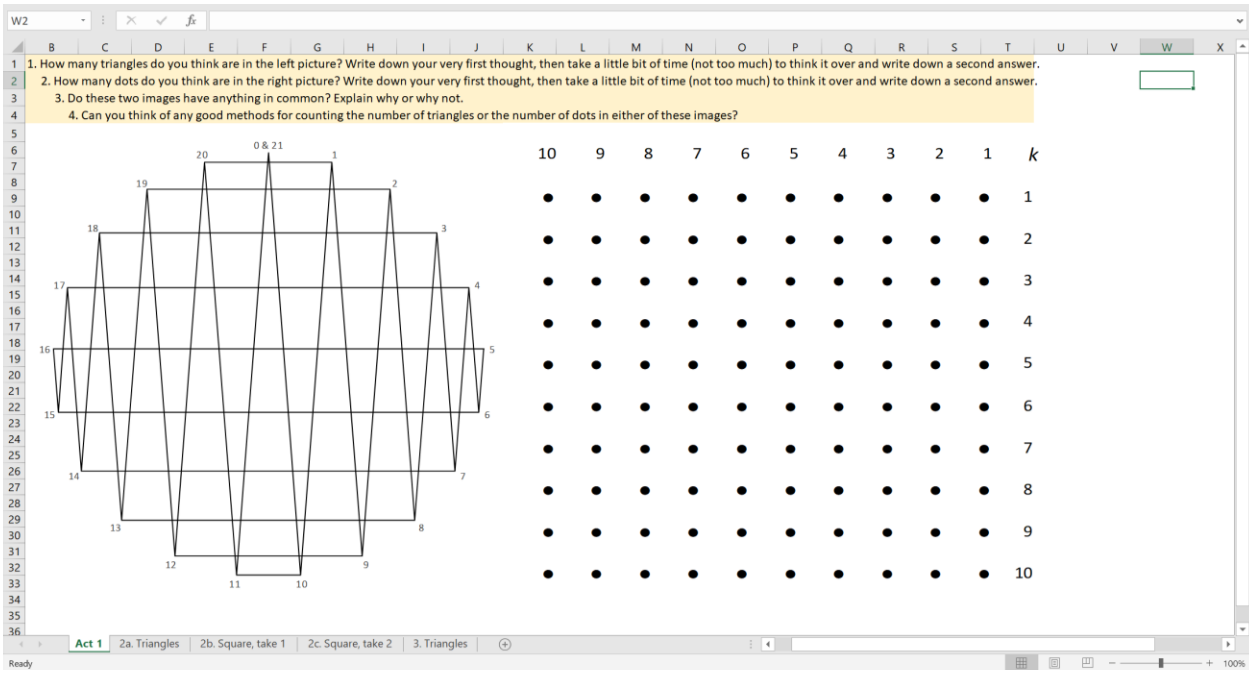
This document is a guide for teachers to use to walk students through the companion Excel file, as well as provides expectations, answers, and explanations to the questions.

ACT ONE

The Excel file opens to a sheet titled **Act 1**. The screen should look like the image below.



1. How many triangles do you think are in the left picture? Write down your very first thought, then take a little bit of time (not too much) to think it over and write down a second answer.

   Students will likely not get this answer right. The point of this question is to intrigue students and get them to consider what the lesson will be about. In actuality, there are 100 triangles.

2. How many dots do you think are in the right picture? Write down your very first thought, then take a little bit of time (not too much) to think it over and write down a second answer.

   All answers to this question are perfectly acceptable. The point of the first question is to show that all answers are valid. For their second answer, some students may immediately realize the image of dots forms a square, and will use multiplication table strategies to count the number of dots. Others may take their best guess by just considering the size of the image. Others still might try to count the number of dots in their second guess. Make sure to limit the time students take on this second question, to avoid this tedious counting.

3. Do these two images have anything in common? Why or why not?

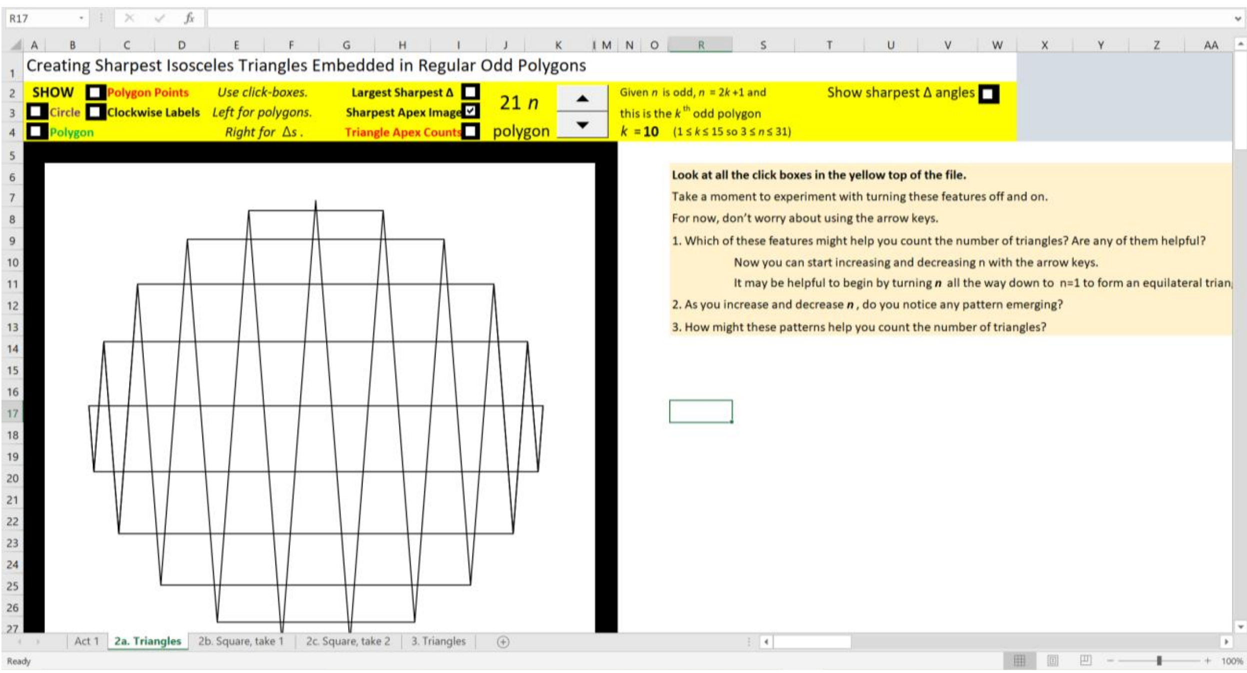
   In actuality, there are 100 dots and 100 triangles. However, students will likely not realize this commonality. If they do give this answer, it is likely just a guess. However, they may give interesting explanations about grids or other such things that promote exploration. If they say they do not have anything in common, you can just hint that they do, and their interest should be piqued.

3. Can you think of any good methods for counting the number of triangles or the number of dots in either of these images?

   The point of this question is to get kids to visualize the dots and triangles in different ways. Some kids, given time to consider this question, may now realize that multiplication would be helpful to calculate the number of dots. Welcome all strategies for counting. Again, this promotes exploratory thinking.

<u>ACT TWO</u>

Have students go to the sheet titled **2a. Triangles**. Their screens should look like this:



Look at all the click boxes in the yellow top of the file. Take a moment to experiment with turning these features off and on. For now, don't worry about using the arrow keys.

1. Which of these features might help you count the number of triangles? Are any of them helpful?

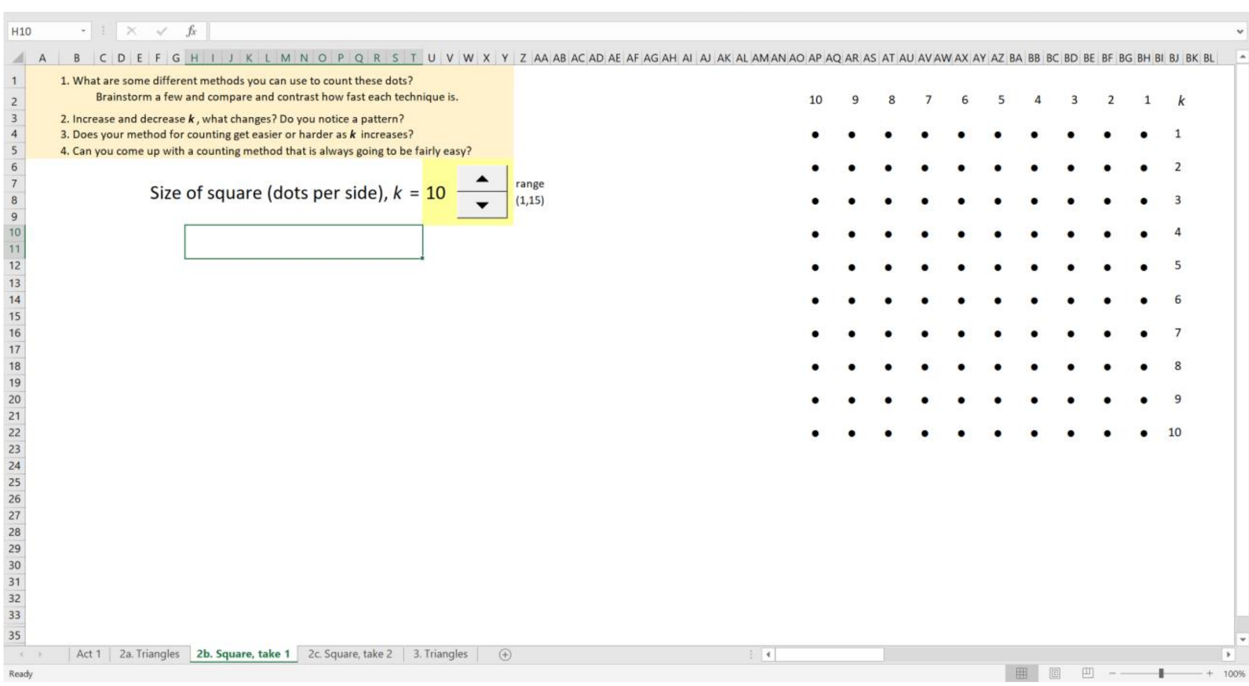
   The most helpful feature for counting triangles is going to be the click box labeled **Triangle Apex Counts**. However, by playing with the other features, students can learn interesting things about the shape. For example, they may see that some angles of the triangles are sharper than others, the degrees of the angles, that it is inscribed within a circle, or that it is even inscribed within a 21-gon, where some of the sides of the triangles make up the sides of the polygon.

Now you can start increasing and decreasing ***n*** with the arrow keys. It may be helpful to begin by turning ***n*** all the way down to 3 to form an equilateral triangle.

2. As you increase and decrease ***n***, do you notice any pattern emerging? How might these patterns help you count the number of triangles?

   For ***n*** = 3, students will easily be able to see the singular triangle. As ***n*** gets increased, they will better notice how triangles overlap with each other to create more triangles. They may even notice that the number of triangles in the center increases linearly, as well as the number of triangles spaced symmetrically around the center triangles.
   If students have **Triangle Apex Counts** toggled on, this could be where they see the "up the hill and back down pattern," but it is unlikely that this will occur as most will move their eyes around the circle instead.

Students should move on to the next page, titled **2b. Square, take 1**. Their screens should look like the image below.



1. What are some different methods you can use to count these dots? Brainstorm a few and compare and contrast how fast each technique is.

   This question gets students thinking about counting and may allow them to realize on their own that multiplication is the fastest way to count a square.

2. Increase and decrease ***k***, what changes? Do you notice a pattern?

   As ***k*** is increased or decreased, the square of dots grows or shrinks. The square of dots is always ***k***×***k*** dots large. Numbers labeling the rows/columns will appear with the dots for students to more easily notice this pattern.

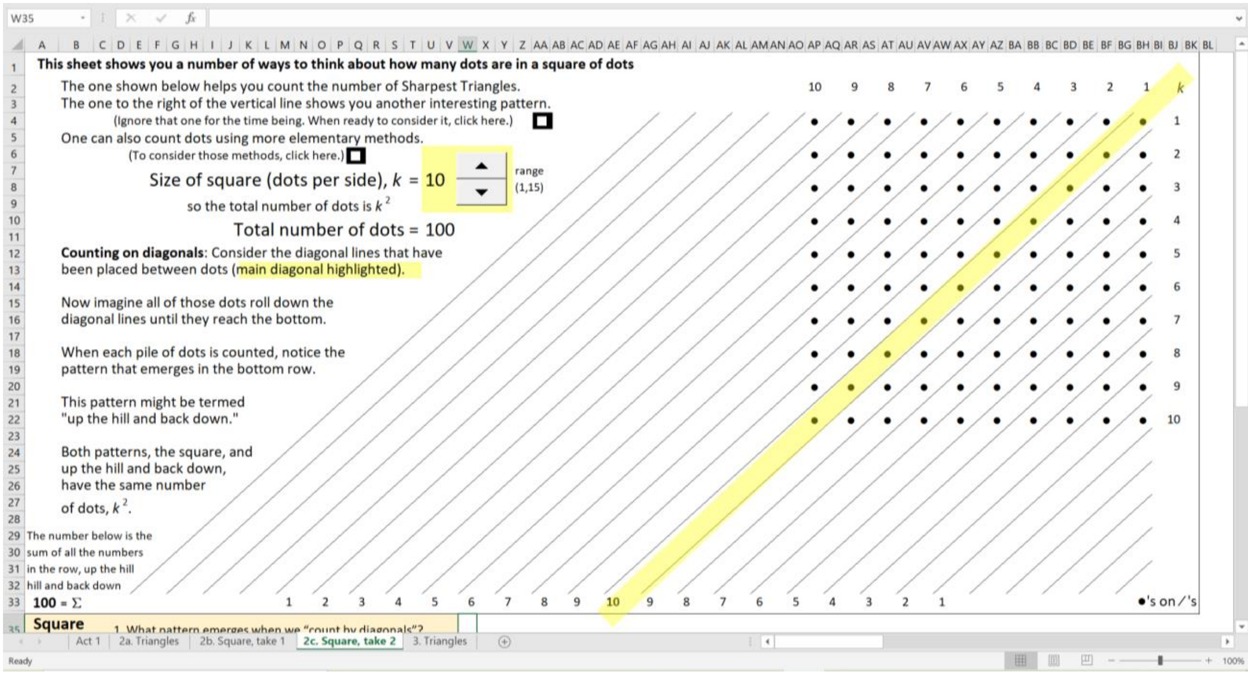
3. Does your method for counting get easier or harder as ***k*** increases?

   Most likely, every student will have more difficulty with counting the dots as ***k*** increases. Students who used multiplication will likely have more trouble computing the size of the square than before once ***k*** increases beyond 10. On the other hand, as ***k*** decreases, other strategies (like individual counting and estimating) for counting the dots will likely become more validated.

4. Can you come up with a counting method that is always going to be fairly easy?

   The students who have now recognized multiplication as a counting method will probably consider it to be fairly easy, given that they can compute the number of dots for any ***k***, provided they have a calculator. If any student has recognized counting by diagonals or other such methods, they will also probably consider it to be fairly easy.

Now direct students to go to the next sheet titled **2c. Square, take 2**. Their screens should look like the image below. Scroll down if necessary to see the discussion questions in the area labeled **Square Take-Aways**.



4. What pattern emerges when we "count by diagonals"?

   The answer is given on the left but it is important for students to articulate it in their own words. We can see that the numbers at the bottom, representing the number of dots in each diagonal, begins at 1, goes all the way up to ***k***, and goes back down to 1 again.

5. What does this pattern tell us about counting?

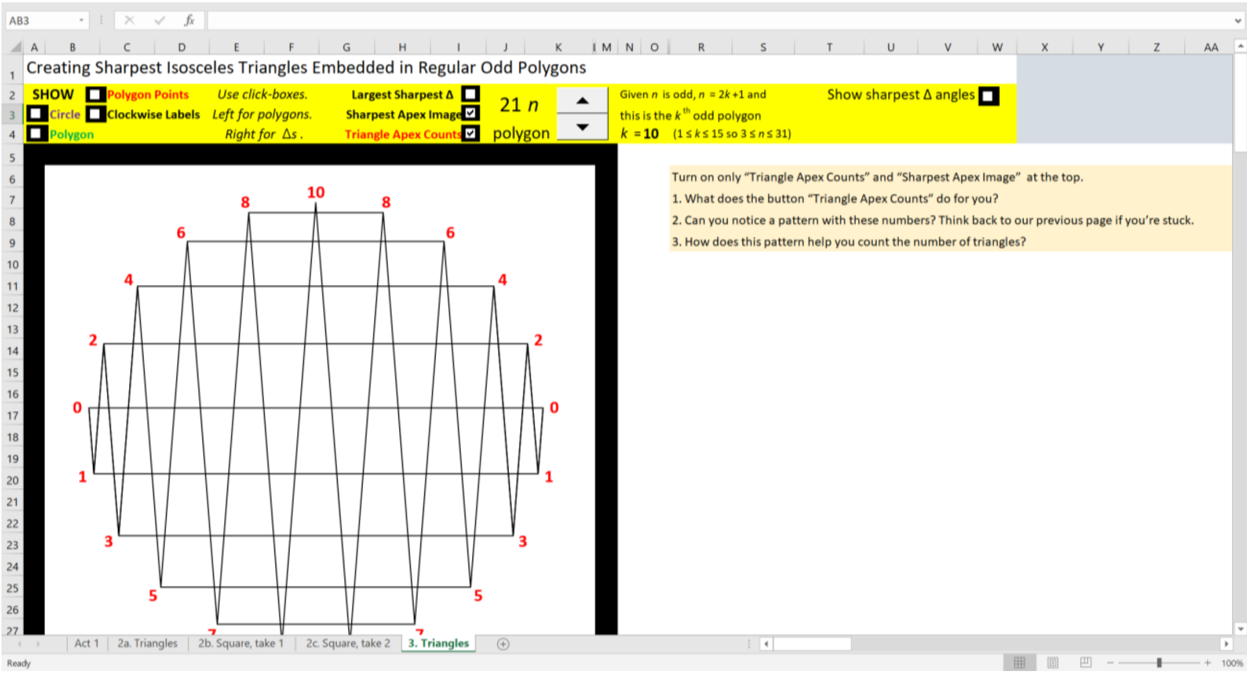
   This pattern tells use that the sum from 1, to ***k***, back down to 1 is equal to $\boldsymbol{k}^2$. In other words, when you add "up the hill and back down" it will always equal the square of the largest number in your "hill".

6. How might it be useful for counting quickly?

   If students cannot come up with an answer to this question, that is okay. What we want to nudge them towards thinking is that if we ever see this "up the hill and back down pattern", we can just multiply the largest number by itself to get the sum of all these numbers. However, any other answers students have for this question should be interesting to hear!

ACT THREE

Finally, now have students open the next sheet titled **3. Triangles**. This is the last page in the lesson. Their screens should look like the image below.



Turn on only **Triangle Apex Counts**” and **Sharpest Apex Image** so your screen looks like the above image.

5. What does the button **Triangle Apex Counts** do for you?

   This button tells the number of triangles that have their apex at each vertex. Students will likely recognize this, but may describe it in different ways.

6. Can you notice a pattern with these numbers? Think back to our previous exercise if you're stuck.

   If students follow a zig-zag pattern, they will now realize the “up the hill and back down” pattern and can tell you that there are $10^2 = 100$ triangles for the 21-gon.

7. How does this pattern help you count the number of triangles?

   Students should now be able to realize (with guidance if needed) that they can use this "up the hill and back down" pattern to immediately calculate the number of triangles. As long as they know how many triangles are in the largest sharpest triangle, they know how many triangles there are total.

   Increase and decrease ***n*** and ask students how many triangles are in each image, as long as they know their squares, they should be able to immediately tell you how many triangles are in the image.

   Take them back to the first page and ask them how many triangles are in the image on the left. Point out to them that indeed, there are the same number of triangles as dots.

   Students typically find this realization fun and may even go home to boast to their siblings and parents that they can immediately tell how many triangles are in an image like this.