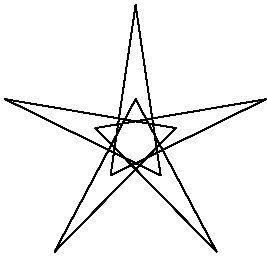
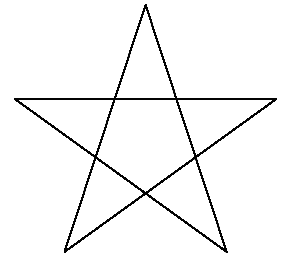


## Searching for Similarity: Examining the *Star-in-a-Star* pattern

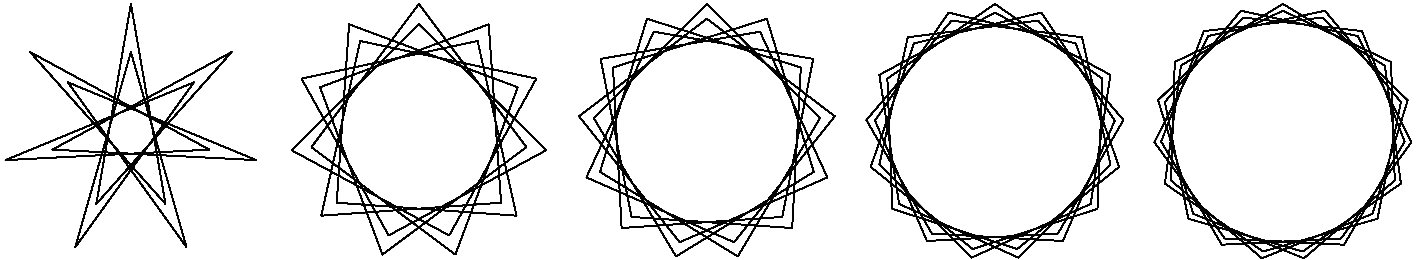


As you begin to play with subdivisions,  $S$ , and points between subdivisions,  $P$ , in file 2 you will undoubtedly encounter the image of a star within a star, shown to the left, given  $S = 2$ ,  $P = 3$ ,  $J = 2$ , and  $n = 5$ , created with 10 lines. Compare that to the traditional star on the right, created with 5 lines.



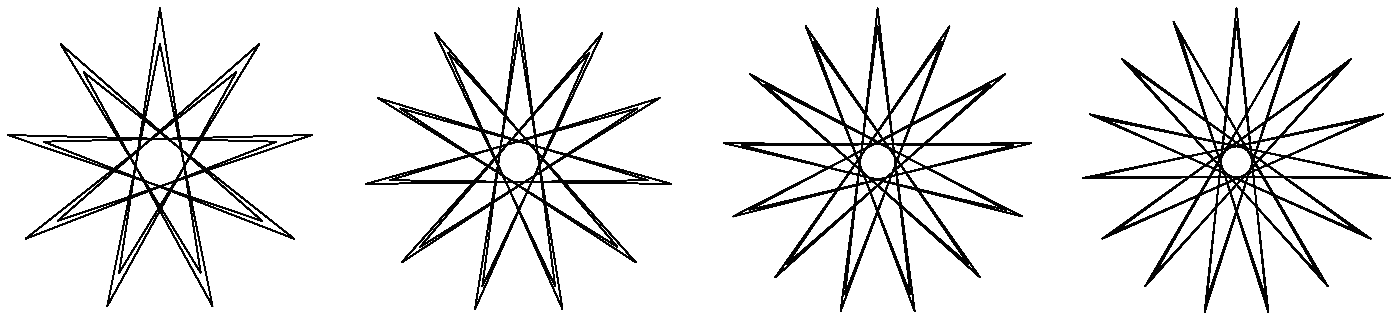
Note that the outer star is more pointed, and the inner star is less pointed, than the star to the right. Additionally, the vertices of the inner star are located at the exact midpoint between every other outer polygonal vertex. Also, the inner star is completely inside the outer.

The question naturally arises: **Are there similar images?** For example, what happens as  $n$  increases holding all else fixed? Interestingly, not all values of  $n$  produce similar images. The next five values,  $n = 7, 11, 13, 17$ , and  $19$ , are:

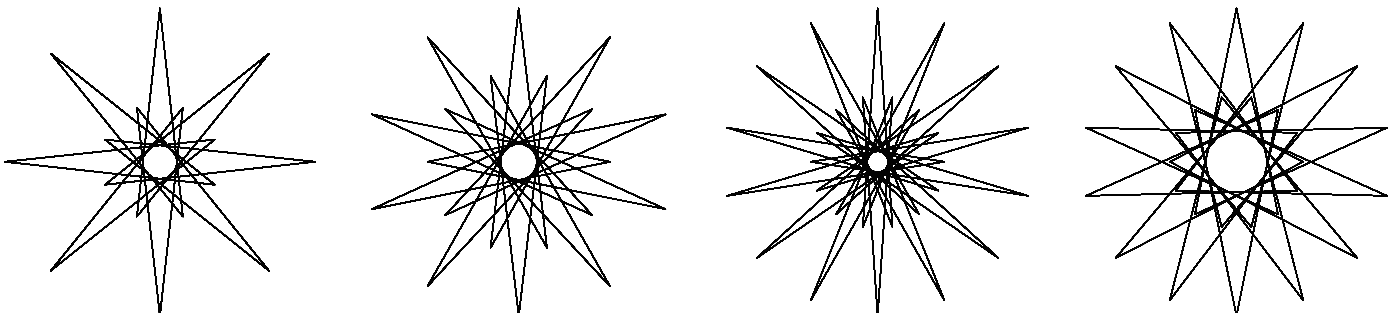


A couple of things are immediately apparent. 1) each  $n$  is odd; 2) not all odd values of  $n$  are represented; and 3) the images have larger and larger centers. Even values of  $n$  reduce the vertices used in the resulting image because  $J = 2$ . Likewise,  $n = 9$  and  $n = 15$  are excluded because they are divisible by  $P = 3$ .

The  $n = 7$  image has a smaller star that is noticeably closer in size to the first than the others because  $P = 3$  is close to half the size of  $n$ . The next row shows 4 odd  $n$  images (9, 11, 13, and 15) that occur when  $P$  increases to maintain this “closest to half” relationship. Note that  $n = 9$  and  $n = 15$  are no longer excluded (because  $P$  is no longer divisible by 3).



The final four images show even values of  $n$ . Inner star points are no longer coincide with outer star point vertices. Instead, they are between outer vertices. This will always be true when  $n$  is even and  $S = 2$ . The first three have inner spikes which are outside the footprint provided by the larger star (unlike all images above) and thus are not formally a “star-in-a-star” but the last one shows that internal even star-in-a-star images are, indeed, possible.



One should note that in all of these images, there are additional regular stars using the closest to center intersections of lines used to create the image. For large  $n$ , these are harder to see due to the size of images on this page.