Objectives:
- Students will be able to (SWBAT) define a vertex and a side in order to develop an understanding about the composition/structure of polygons.
- SWBAT observe similarities and differences among polygons up to 15 sides in order to create and explain patterns among vertices and sides.
- SWBAT identify common polygons in order to compare and contrast the defining attributes rather than non-defining attributes.
- SWBAT identify common polygons by the number of sides and vertices in order to describe real objects in the environment using correct names.

Standards:
- CCSS.MATH.CONTENT.K.G.A.1 - Describe objects in the environment using names or shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
- CCSS.MATH.CONTENT.K.G.A.2 - Correctly name shapes regardless of their orientations or overall size.
- CCSS.MATH.CONTENT.K.G.B.4 - Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g. number of sides and vertices/"corners") and other attributes (e.g. having sides of equal length).
- CCSS. MATH.CONTENT.1.G.A.1 - Distinguish between defining attributes (e.g. triangles are closed and thee-sided) versus non-defining attributes (e.g. color, orientation, overall size); build and draw shapes to possess defining attributes.
- CCSS.MATH.CONTENT.2.G.A.1 - Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Introduction Lesson:
Use the Excel worksheet with these discussion questions. *Introduce points and vertices.*

Questions for shapes 3-15:

1. What does 1 look like? *(A point or vertex)*
2. What does 2 look like? *(A line segment between 2 points/vertices or a side)*
3. *Looking at a triangle, identify the points/vertices and the lines/sides. Note that the sides and vertices create angles.*
4. *Note additionally that all sides are the same length -- they are equal.*
5. What changes when you move the arrows up and down? *(Sides and points/vertices)*
6. What is the relationship between the number of sides and number of points? *(Same)*
7. What point doesn't move when changing these shapes (T,B,R, or L)? *(Top)*
   a. T (top), B (bottom), R (right), L (left)
8. As we increase the number of sides, are the points getting closer together or farther apart?

9. When there are 3 sides (a triangle), there is a flat bottom at B. When there are four sides (a square), there is a pointy bottom at B. When there are five sides (a pentagon), there is a flat bottom. When there are six sides (a hexagon), there is another pointy bottom! What comes next in the pattern?!? *(Flat, pointy, flat, pointy)*
10. Can you state a general pattern (or rule) for when you will have a pointy bottom? *(Even numbers of vertices will have pointy bottoms.)*
11. For what numbers are the right and left sides pointy at R and L? *(4, 8, 12)*
12. For what numbers are the right and left sides vertical (not slanted)? *(6, 10, 14)*
13. For what numbers are the sides at R and L slanting towards the top? *(3, 7, 11)*
14. For what numbers are the sides at R and L slanting towards the bottom? *(5, 9, 13)*

Additional student exploration activity/discussion:

15. NOW go to number 20! Let's make a prediction - if we keep making the number bigger, what shape will it look like at 50? *(A circle)*
16. *Teachers can point out that this polygon with 50 vertices is pointy at B and vertical at R and L, just as predicted by earlier patterns.*

Teachers may want to make a note about how it is NOT a circle, it has many sides that are nearing a circle but will never become one.

Follow up Activity: Worksheet #1 (The last 6 pages of this file are prepared worksheets 1-6.)

COUNTING USING THE STARS SHEET
Objectives:

- SWBAT accurately count and use 1:1 correspondence in order to connect vertices with jumps to draw a variety of stars.
- SWBAT define a vertex and a jump in order to work with stars with more than five vertices and a varying number of jumps.
- SWBAT connect multiple vertices by using multiple jumps in order to draw a variety of stars.
- SWBAT make connections between vertices and jumps in order to understand patterns between stars that create the same shape, straight lines, polygons, etc.

Standards:
Only Kindergarten standards for Counting and Cardinality

- CCSS.MATH.CONTENT.K.CC.B.4 - Understand the relationship between numbers and quantities; connect counting to cardinality.
- CCSS.MATH.CONTENT.K.CC.B.4.A - When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- CCSS.MATH.CONTENT.K.CC.B.4.B - Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
- CCSS.MATH.CONTENT.K.CC.C.6 - Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g. by using matching and counting strategies.
- CCSS.MATH.CONTENT.K.G.B.4 - Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g. number of sides and vertices/"corners") and other attributes (e.g. having sides of equal length).

Lessons based on Stars Sheet:
Lesson 1
Launch -

- How to Draw a Star Activity:
  - Teachers will split the students into partners or small groups. Ask students to draw a star. Students within groups will be able to discuss how they draw a star. If a student does not know how to draw a star, other peers can show them.
    - Teachers should be checking in with groups to modify and adapt activity.
    - Teachers can offer partners and groups to share how they drew their star.
  - After providing the students with a couple minutes to draw a star, the teacher will demonstrate how to draw a star using vertices and jumps in the following step.
- *Using the video explanation*, teachers will demonstrate to students how to draw a star by drawing vertices and using jumps.
  - Key points:

- Be specific to connect knowledge to vertices.
- Explain jumps using 2-3 jumps as examples
- Differentiation: Challenge students who already know to draw a different way

Activity -

- Teachers should introduce students to different kinds of stars with more vertices than five.
- **Worksheet #2: Connect the Dots**
- Once students complete the warm-up worksheet, students will work with the Excel file. Fix the number of jumps to 3. They will specifically work with changing the number of vertices. Use the discussion questions below to guide students when using the Excel file.

Discussion Questions -

1. What shape is created when the number of vertices is 5? *(A regular star.)*
2. What shape is created when the number of vertices is 6? Why could this be? *(A line segment.) (Allow students to brainstorm possible reasons. They do not have to be correct. They do not have to understand the actual reason yet in this lesson.)*
3. What shape is created when the number of vertices is 9? Why could this be? *(A triangle.)*
4. What shape is created when the number of vertices is 12? Why could this be? *(A square.)*
5. As the number of vertices increase, what patterns do you notice?
   a. *(The lines intersect further and further away from the middle of the star.)*
   b. *(There is a pattern of star, star, polygon.)*
   c. *(The polygons increase by one side each time in the pattern.)*
6. As we increase the number of vertices, what shape do the stars and polygons begin to look like? *(A circle.)*

Formative Assessment -

- Worksheet #2 and discussion questions can be utilized as a formative assessment. Other exit tickets can be made according to the teacher's perspective.

Lesson 2
Launch -

- Teachers should start the lesson off by explaining how to count jumps by using a mnemonic device. An example is provided:
  ○ Jumps should look like a puffy cloud in the sky. (Students can remember clouds are in the sky, just like the stars.)
  ○ Explain that the jump moves in a circular motion on the outside of the circle (the vertices)
    ■ Teachers should explain the difference between jumps and line segments.
- Number talk:

○ Teachers will split students into four groups. Each group will draw a 5-pointed star with a different number of jumps. (7, 8, 12, and 13 jumps)
○ Have the students share the figures they have created. They will all look like the same star. Students should brainstorm possible reasons.

Activity -
● Students will be working with a different number of jumps to draw stars. The number of vertices will remain the same. For this lesson, we will stick with five vertices. Teachers should touch on the idea that different numbers of jumps create different patterns depending on the number of vertices in the star.
   ○ *Sometimes, the final image looks the same (as with a 5-pointed star where there is only ONE final image, despite how many jumpers occurred in getting there). If 7, 8, 12, and 13 jumps had been drawn using 17 vertices, each would have produced different images.*
● **Worksheet #3: Star Jumps**
   ○ This worksheet is similar to the number talk, but the purpose is to solidify the objective of the number talk.
● Once students complete the worksheet, students will work with the Excel file. They will specifically work with changing the number of jumps. Use the discussion questions below to guide students when using the Excel file.

Discussion Questions -

1. How is today's lesson similar to yesterday's lesson? What patterns and shapes do you remember?
2. Let's imagine 13 vertices equally distributed around a circle. What shape is created when the number of jumps is 1? Why could this be? *(A polygon with 13 sides.) (Allow students to brainstorm possible reasons. They do not have to be correct. They do not have to understand the actual reason yet in this lesson.)*
3. What shape is created when the number of jumps is 12? *(A polygon with 13 sides again.)*
4. What shape is created when the number of jumps is 13? Why could this be? *(A point.) (Allow students to brainstorm possible reasons. They do not have to be correct. They do not have to understand the actual reason yet in this lesson.)*
5. What do you notice about the stars that are created when the number of vertices are 6 and 7? Why could this be? *(They are the same.)*
6. What other patterns do you notice throughout the lesson?
   a. *Students may notice that numbers opposite of the median create the same shape. For example, 6 and 7, 5 and 8, 1 and 12, etc. If they do, ask whether they see the general pattern (or rule). The complex pattern is: J and non-J produce the same image. Put another way the sum of the jumps is (a multiple of) n. (J + n-J = n)*
   b. *This question is open for student observations.*

Exit ticket -
● **Worksheet #4: Exit Ticket 1** - Find 1 more number of jumps that creates a five sided star, other than 2, 3, 7,8 12, and 13. *Exit ticket is a half sheet per student.*

Lesson 3
Launch -
- All students in the class will form a circle. *Teachers may need to move furniture in order to accommodate space for the activity.* Any number of students works for this activity. Teachers will need a large roll of yarn. Introduce the activity by challenging the students to make a huge star. Explain to the students that they are the vertices of the star. Teacher will instruct students to jump by two vertices.
    - If there are an even number of students - half of the students will be left out of the star.
    - If there are an odd number of students - all students will be included.
- Repeat the activity. This time the teacher will step into the circle to add one more person. This will reverse the outcomes from the first activity.
    - The teacher will change the number from even to odd or vice versa.
- The goal of these activities is to create purposeful mistakes for incidental teaching and student discovery. Examples of purposeful mistakes are as follows:
    - Take one jump between students. *Notice that the pattern makes a polygon.* Reinforce this concept by using 4-6 students.
    - Jump the same amount as the number of students. (Example - 15 students, 15 jumps) *Notice that the pattern makes a point.*
    - Take the number of students (needs to be an even number) and divide it in half. This is the number of jumps you will do. *Notice that the star turns into a straight line (line segment).*
    - Students can pick their own number of vertices and number of jumps to make more discoveries and patterns on their own. Teachers can split students into smaller groups if needed.

Activity -
- Students will work with the Excel file. Teachers should explain that the activity with yarn represented what you can do on Excel. Use the Excel file. After a couple minutes of independently changing the vertices and jumps, the teacher can lead some discoveries with discussion questions.
- *Have students choose a number of possible vertices and answer the following questions.*

Discussion Questions -

1. Can you ever make the same shape twice using a different number of jumps but the same number of vertices? *(May be review from yesterday's last discussion question. This is a key point for students to connect their knowledge from Lesson 2.)*
2. If you have an even number of vertices, when do you make a line segment? *(When the number of jumps is half of the number of vertices.)*
3. When do the shapes disappear and make a point? *(When the number of jumps equal the number of vertices.)*

4. Can you ever make the same shape twice using a different number of jumps and different number of vertices? *(Open ended for student exploration.)*
5. Who can make the pointiest star? *(Open ended for student exploration.)*

Exit Ticket -
- Answer two of the following questions in relation to the class discussion:
  - How is today's lesson similar to our past two lessons? What connections can you make?
  - What patterns do you notice throughout the lesson?
  - Was there anything difficult to understand?

HIGHER ORDER THINKING FOLLOW UP MINI LESSONS:

Addition/Multiplication/Division:

- You can incorporate addition with the number of jumps. *For example, if you start at Point 0 and you must jump by 2, you would connect Point 0 to Point 2 because 0 + 2 = 2. The next jump will go to Point 4 then Point 6 (which is the same as Point 1) and Point 8 (which is the same as Point 3) and Point 10 (which is the same as Point 5 and Point 0). This is just counting by twos.* Two worksheets are attached to practice this concept.
    - See **Worksheets #5 and #6** for supplemental activities to Excel file
- You have a polygon with 15 sides. If there are 15 vertices, how can we make a triangle or a pentagon by changing the number of jumps from 1?
    - This kind of question involves repeated addition, multiplication, and division. This can be used to the teacher's discretion.

Additional Complex Patterns:

- If you have a polygon with an even number of vertices, how could you make a line segment by changing the number of jumps?
    - Answer: Divide the even number in half.
- Notice what happens when you change the number of jumps of an even-sided polygon to the median. *(For example, if you have an octagon, what happens when you change the number of jumps to 4?)* Now, notice what happens when you change the number of jumps to numbers 1 away from the median. *(For example, if you have an octagon, what happens when you change the number of jumps to 3 and 5?)*
    - Answer: Reinforce the idea that the median creates a line segment.
    - Answer: Numbers on either side of the median create the same shape and the two numbers add to the number of vertices in the shape.
- Notice what happens when you change the number of jumps of an odd-sided polygon to numbers on either side of the median. *(For example, if you have a nonagon, what happens when you change the number of jumps to 4 and 5?)*
    - Answer: Numbers on either side of the median create the same shape.
- Notice all the following patterns on opposite sides of the median of odd- and even-sided polygons. *(For example, if you have a decagon, what happens when you change the number of jumps to 5? Then to 4 and 6? Then to 3 and 7? Then to 2 and 8? Then to 1 and 9?)*
    - Answer: The pattern of the shapes mirror each other. Each pair creates the same final image.
    - Answer: You can connect addition to this mirroring pattern. Each pair, when added together, equals the total number of sides of the polygon. For example, 4 + 6 = 10 and 3 + 7 = 10.
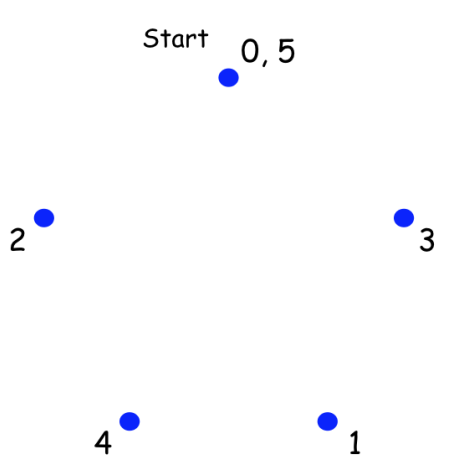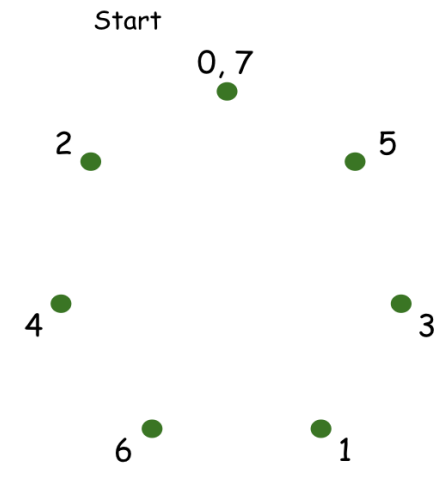
Worksheet #1: Polygon Basics

Directions: Using the Excel online program, create the polygon or shape that looks the closest to each real life example. *(There can be multiple correct answers.)*
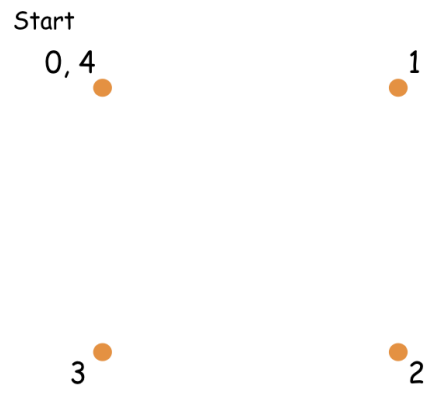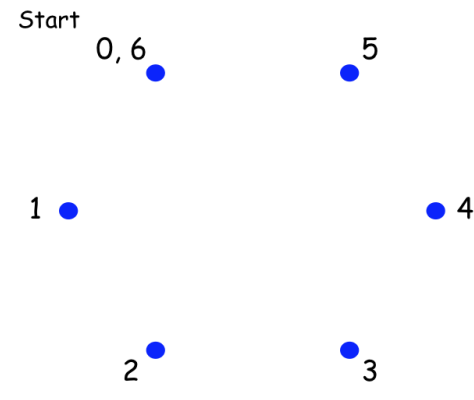
1.

2.

3.

4.

5.

6.

7.

8.
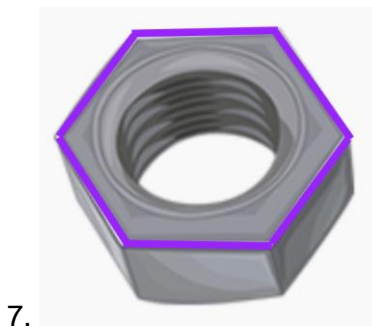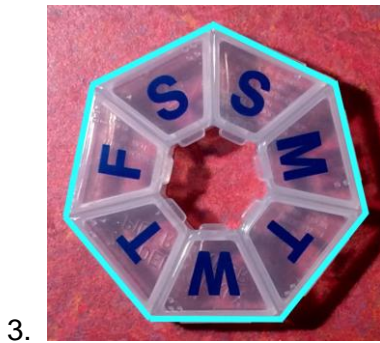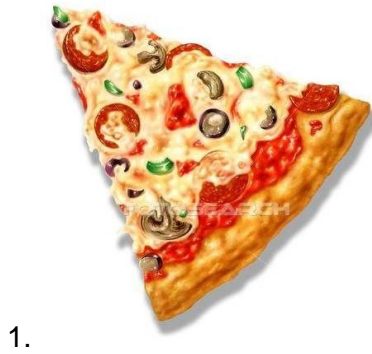
Worksheet #2: Connect the Dots

Directions: Practice drawing different stars! Connect the dots in numerical order.

1)

Start

0, 5

2

3
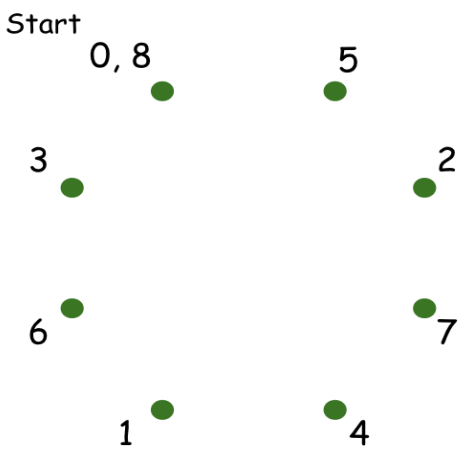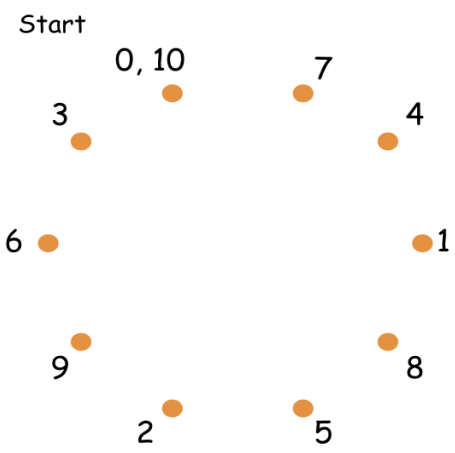
4

1

2)

Start

0, 7

2

5

4

3

6

1

3)

Start

0, 4

1

3

2

4)

Start

0, 6

5

1

4

2

3

5)

Start

0, 8

5

3

2

6

7

1

4

6)

Start

0, 10

3

7

4

6

1

9

8

2

5

Worksheet #3: Star Jumps

Directions: Practice drawing your stars! Count the correct number of jumps between points in order to connect the dots and make a star.

Example:
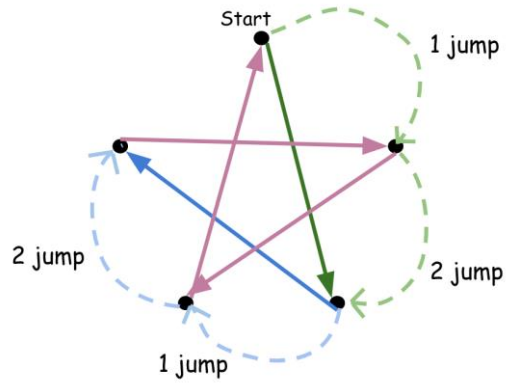
1. Draw a star with 3 jumps:
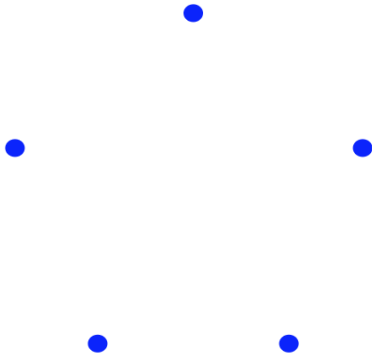
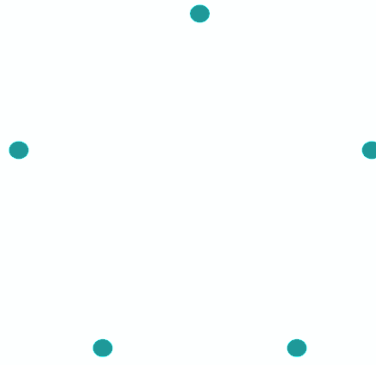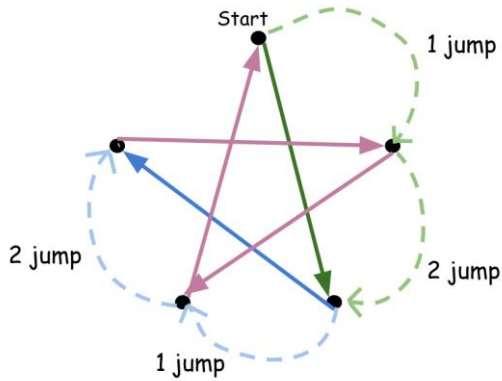2. Draw a star with 7 jumps:
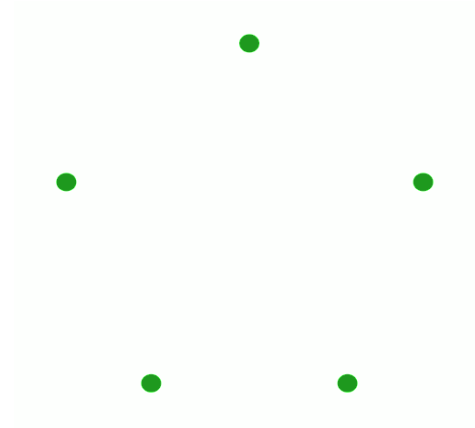
3. Draw a star with 8 jumps:
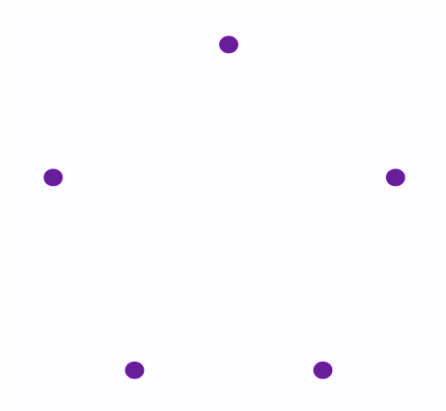
4. Draw a star with 12 jumps:
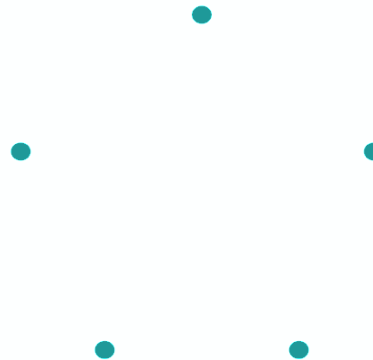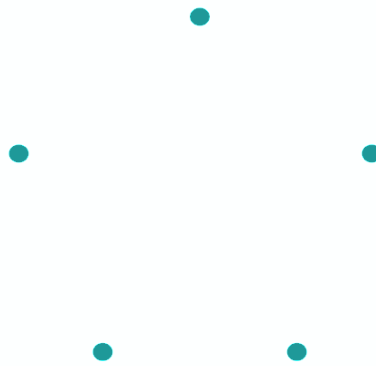
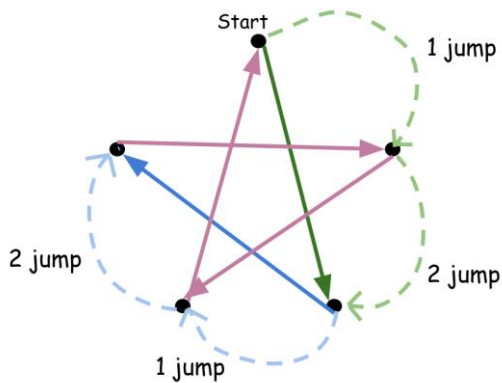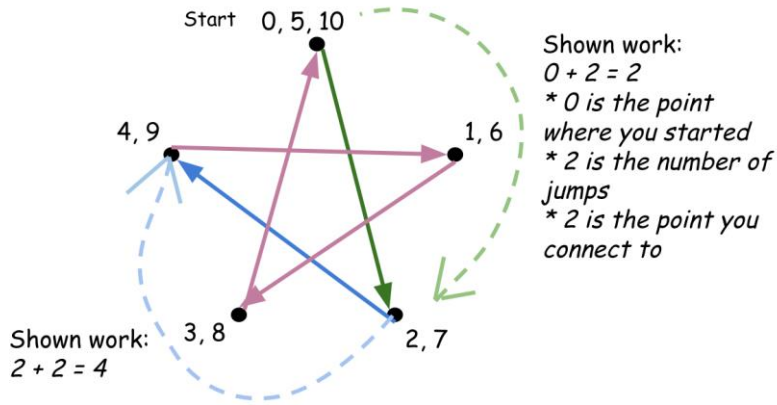## Worksheet #4: Exit Ticket for Star Lesson 2

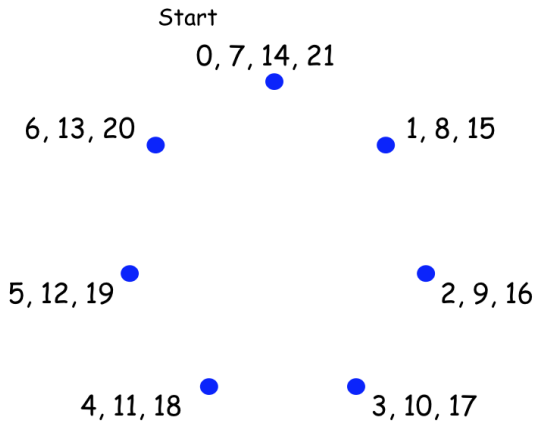**Directions:** Find one more number of jumps that create a five sided star.
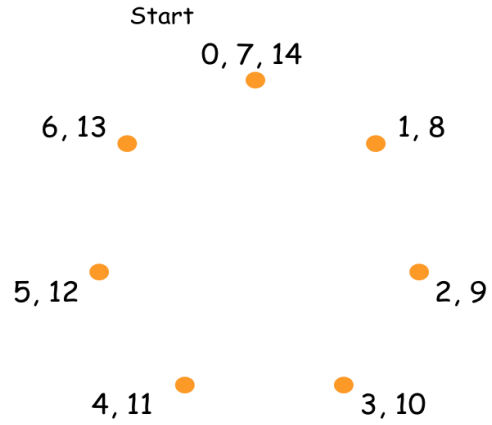
Show work here:

Worksheet #5: Addition Part 1

Directions: Add the correct number of jumps between points in order to connect the dots and make a star. Do NOT connect the dots in numerical order. The numbers represent the sum after adding the number of jumps. An example is provided.
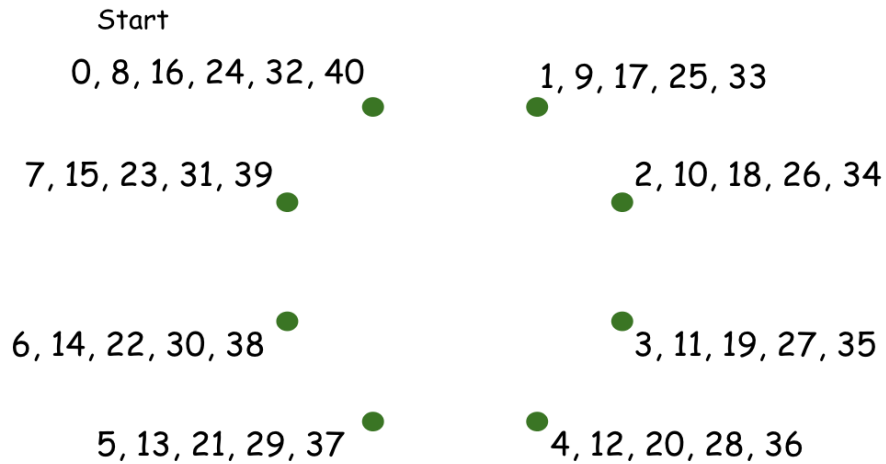
Start   0, 5, 10

4, 9   1, 6

3, 8   2, 7

Shown work:
$0 + 2 = 2$
* 0 is the point
where you started
* 2 is the number of
jumps
* 2 is the point you
connect to

Shown work:
$2 + 2 = 4$

1. Draw a star with 3 jumps:

Start
0, 7, 14, 21

6, 13, 20   1, 8, 15

5, 12, 19   2, 9, 16

4, 11, 18   3, 10, 17

2. Draw a star with 2 jumps:

Start
0, 7, 14

6, 13   1, 8

5, 12   2, 9

4, 11   3, 10

3. Draw a star with 5 jumps:

Start

0, 8, 16, 24, 32, 40   1, 9, 17, 25, 33

7, 15, 23, 31, 39   2, 10, 18, 26, 34

6, 14, 22, 30, 38   3, 11, 19, 27, 35

5, 13, 21, 29, 37   4, 12, 20, 28, 36
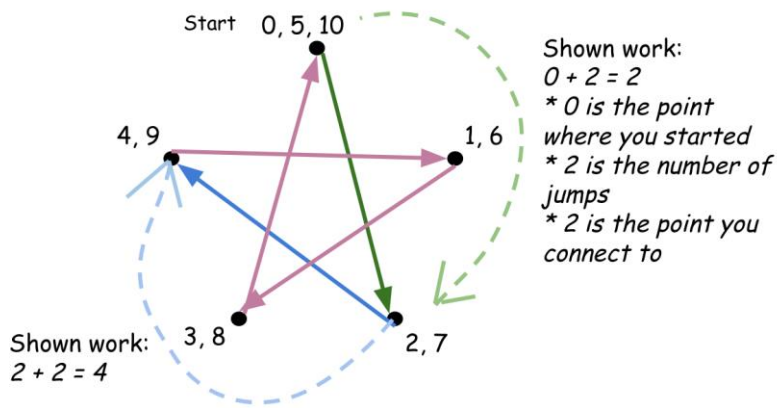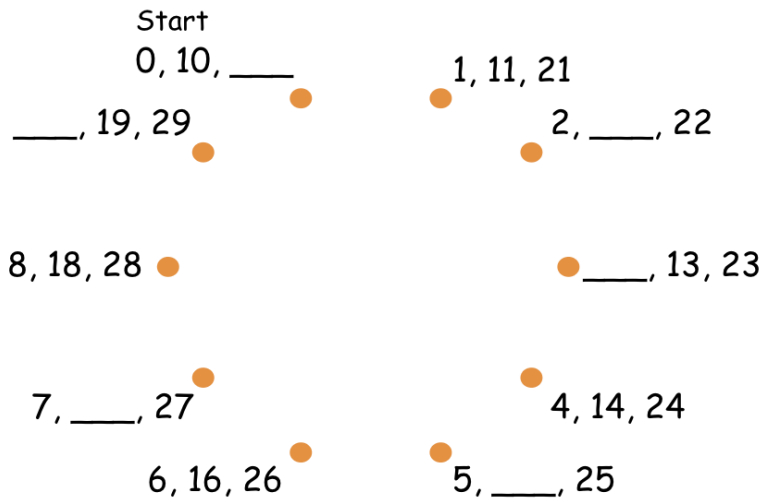
Worksheet #6: Addition Part 2

Directions: Add the correct number of jumps between points in order to connect the dots and make a star. This time, you may have to fill in some of the correct numbers.

Start 0, 5, 10

Shown work:
0 + 2 = 2
* 0 is the point where you started
* 2 is the number of jumps
* 2 is the point you connect to

4, 9

1, 6

Shown work:
2 + 2 = 4

3, 8

2, 7

1. Draw a star with 3 jumps:

Start
0, 10, ____

1, 11, 21

____, 19, 29

2, ____, 22

8, 18, 28

____, 13, 23

7, ____, 27

4, 14, 24

6, 16, 26

5, ____, 25

2. Draw a star with 2 jumps:

Start 0, 5, ____

4, 9

1, ____

3, ____

____, 7