2. Common Core Standards Met by Grade

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The following grade level standards are pulled from the national Common Core State Standards for Mathematics [1]. Statements from CCSS are denoted in italics, with explanation of how this standard is met is noted in regular type. (Where appropriate, specific documents are specified in parentheses.)

Kindergarten

- (K.MD.1) "*Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.*"
    - This standard is the very foundation of File 2. Each image has 4 measurable attributes, ***S***, ***P***, ***J*** and ***n***. Two of those attributes, ***J*** and ***n***, were introduced in File 1. (See 2.1)
- (K.MD.2) "*Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference.*"
    - Images differ from each other depending on whether ***S*** > ***P*** versus ***S*** < ***P***. (See 2.2b)

First Grade

- (1.G.1) "*Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.*"
    - Students may recognize how size of polygons and length of their sides are non-defining attributes to some shapes. For example, they should realize that the images made by the ***S, P, J, n*** values: 1, 1, 1, 5; 8, 3, 1, 3; and 5, 22, 3, 11 are all pentagons, despite looking different and being made in different ways. (See 2.5c and 2.9a)
- (1.G.3) "*Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.*"
    - Students may click "Show vertices and labels" to consider which vertices they would need to draw lines between to divide shapes into halves and fourths. This may lead to a deeper appreciation of shapes, as students recognize which shapes can and cannot be divided into fourths, as they grapple with which vertices they should connect to divide the shapes. By connecting labeled, numbered vertices, students may also plant roots for a learning of multiplication and division.

Second Grade

- (2.G.1) "*Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.*"
    - Students may be instructed to find an image with 11 equal angles, for example, and can appreciate the difference between number of vertices and number of angles (an ***n*** = 11 star will have many other angles besides the ones at vertex points). They may also be

instructed to find more complex images, such as one that contains both a hexagon and a triangle (like image made by the ***S, P, J, n*** of 3, 2, 1 ,3).

- (2.G.3) "*Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.*"
    - Just as students clicked "Show vertices and labels" to divide shapes into halves and fourths, they may do the same for thirds. Using labeled vertices should allow students to understand and describe the division of shapes into thirds easily, as it provides structure to their divisions. Furthermore, subdivisions divide line segments into equal parts, which reinforces the idea of division into equal shares. (See 2.5)

Third Grade

- (3.OA.3) "*Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.*"
    - As students become familiar with the file, they will start to recognize how MOD arithmetic creates images. File 2 can demonstrate 'leftovers' from division visually. Students must critically think about multiplication, division, leftovers, and common divisors to understand how their shapes were created.
- (3.OA.9) "*Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*."
    - The very nature of PwP is recognizing arithmetic patterns in an intuitive form: visually. As students create images with larger numbers for their variables, they will recognize many themes of common divisors, recurring images, and similar attributes of images which should all be analyzed through a lens of arithmetic patterns.
- (3.NF.3b) "*Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.*"
    - PwP is a fantastic visual fraction model. Students can recognize why a ***J, n*** = 6, 15 star looks the same as a ***J, n*** = 2, 5 star (because 6/15 = 2/5). Students should also recognize how to simplify their variables, to recreate their images. All of this stems from an understanding of greatest common divisors. (See 2.4)
- (3.G.1) "*Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.*"
    - While not always specifically referring to quadrilaterals or standard shapes, students should become familiar with classification of shapes as they explore PwP. Documents in 2.8 "Image Archetypes," explain how certain images, while created differently, actually share enough characteristics to be classified together.

Fourth Grade

- (4.OA.4) "*Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.*"
    - Recognizing factors and multiples is an important skill throughout PwP. Students should be aware of all the factors of a whole number as they use File 2, to create images with a desired number of vertices, or levels, etc. They will naturally gain an awareness for common factors by simply recognizing visual patterns.
- (4.OA.5) "*Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*"
    - Obviously, generating shape patterns is the very core of PwP. Furthermore, all of these shape patterns are actually number patterns (the numbers being the manipulated variables). By recognizing shape patterns, students will also learn number patterns. For example, students may learn the even-odd pattern described in the CC standard by examining the bottom of a polygon: the bottom is flat when ***n*** is odd, and pointed when ***n*** is even.
- (4.NBT.6) "*Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.*"
    - As explained earlier, an appreciation for MOD calculations is important for PwP. When creating stars, students will regularly be working with whole-number remainders.
- (4.MD.5) "*Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement*…"
    - Many challenge questions force users to explore the angles of the stars and polygons they are creating. Students may be asked to explain why the apex angle of their non-regular polygon is smaller than that of a regular polygon, or calculate various degrees of angles in their stars.
- (4.G.1) "*Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.*"
    - By playing with File 2, students can draw line segments, angles, perpendicular lines, and parallel lines. Students may find patterns for creating different types of lines and angles. Additionally, they will intuit the differences between obtuse, right, and acute angles by playing with the file and recognizing how images form around these types of angles.
- (4.G.3) "*Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.*"

- Lines of symmetry can be discussed for any image made in File 2. Every image has at least one (vertical) line of symmetry, and different types of stars and polygons and more complex images have varying numbers of lines of symmetry. (See 2.5)

Fifth Grade

- (5.OA.3) "*Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*"
  - Numerical patterns will oftentimes overlap with each other in File 2 and can/should be analyzed in conjunction. For example; a 1, 1, 2, 8 image and a 1, 2, 1, 8 image both look identical to a 1, 1, 1, 4 polygon due to different patterns, one between ***S*** and ***P*** and another between ***J*** and ***n***.
- (5.G.3) "*Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*"
  - By analyzing regular and non-regular polygons, which is a comparison performed in many challenge questions, students should build foundational knowledge of categories and sub-categories of two-dimensional figures.

Sixth Grade

- (6.NS.4) "*Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.*"
  - Greatest common factor is referred to as GCD (greatest common divisor) extensively throughout File 2 documents and is best recognized when working to simplify or recreate images.
- (6.EE.2) "*Write, read, and evaluate expressions in which letters stand for numbers.*"
  - PwP provides lots of pre-algebraic reasoning opportunities for students. Most obviously, images are created through manipulation of different variables in which letters stand for numbers. Many documents also say things like "If you want ***k*** circles, set ***S*** = 2***k***" or "Set ***n*** = ***P*** = ***S***-1 or ***S***+1. Here, ***n*** = ***P*** = 39." These are all expressions that generalize numbers into letters.
- (6.EE.4) "*Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).*"
  - This is a skill that may be indirectly worked with File 2. Students will likely transition from describing patterns additively to describing them multiplicatively as they gain familiarity with PwP.
- (6.EE.5) "*Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.*"

- Students are commonly asked to recreate images, or specify the values that created an image, in challenge questions. This is not asking 'what values make the equation true?' but rather, 'what values make the image true?' and promotes even greater critical thinking.

- (6.EE.6) "*Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.*"
  - After using File 2 for a while, students will become increasingly comfortable with representing numbers and analyzing expressions that use ***S***, ***P***, ***J***, and ***n***.
- (6.EE.7) "*Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.*"
  - File 2 documents regularly use language like this to understand patterns that occur, and to describe how certain images are created. For example, the 2.8 Quivering Polygons document writes "***S*** = ***d*******J***" after specifying what ***d*** represents, of course.
- (6.EE.8) "*Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem.*"
  - File 2 documents also commonly use language like this. For example, the first Lines of Symmetry document utilizes the language of inequalities as restraints by writing "a line of symmetry may only be drawn between two vertices ***k*** and $\boldsymbol{n}/\ \mathbf{2} + \boldsymbol{k}$, where ***k*** is any integer less than ***n***/2." Other documents explicitly use inequalities as restraints, like the third 2.8 Stacked Circles document that says "for even ***n*** and for ***J*** < (***n***-1)/2."
- (6.G.1) "*Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.*"
  - The Area Challenge Questions for File 2 ask students to find the areas of various images and explicitly challenges them to "cut apart an image and reorganize the pieces" to find the area of more complex polygons.

Seventh Grade

- (7.EE.4a) "*Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers.*"
  - Students may be asked, in challenge questions, to create an image with a certain number of vertices and a desired number of line segments between vertices. Students will need to translate these complex demands into ***S***, ***P***, ***J***, ***n*** manipulations.
- (7.G.2) "*Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.*"
  - Students are commonly challenged to draw geometric shapes under certain conditions in File 2. Challenge questions could certainly ask students to create stars that contain no triangles (which is rare) or to contain other shapes, like hexagons or octagons. Some challenge questions also specify the value of one variable (e.g. ***n*** = 8) and ask students to create non-regular specific polygons with that value.
- (7.G.5) "*Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.*"

- The Brunes Star Challenge Questions, for instance, require students to analyze related angles to find more angles, building out an understanding of all the angles in the image. Ultimately, students use all this knowledge to calculate the area of certain parts of the star.

Eighth Grade

- (8.G.1) "*Verify experimentally the properties of rotations, reflections, and translations.*"
  - Reflection is discussed in the 2.5 symmetry documents since lines of symmetry are understood as the line over which a polygon can identically reflect. Rotation is also discussed in File 2.5c symmetry document. Users are asked to imagine rotating the image to different vertices, and explain the result in terms of symmetry.
- (8.G.7) "*Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.*"
  - In the Brunes Star Challenge Questions document, for example, users are told "to determine the coordinates of vertex 12 (at the intersection of line (1,11) and (13,7)). Next, use the Pythagorean Theorem to show that the sides are in the ratio 3-4-5."

References

[1] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.