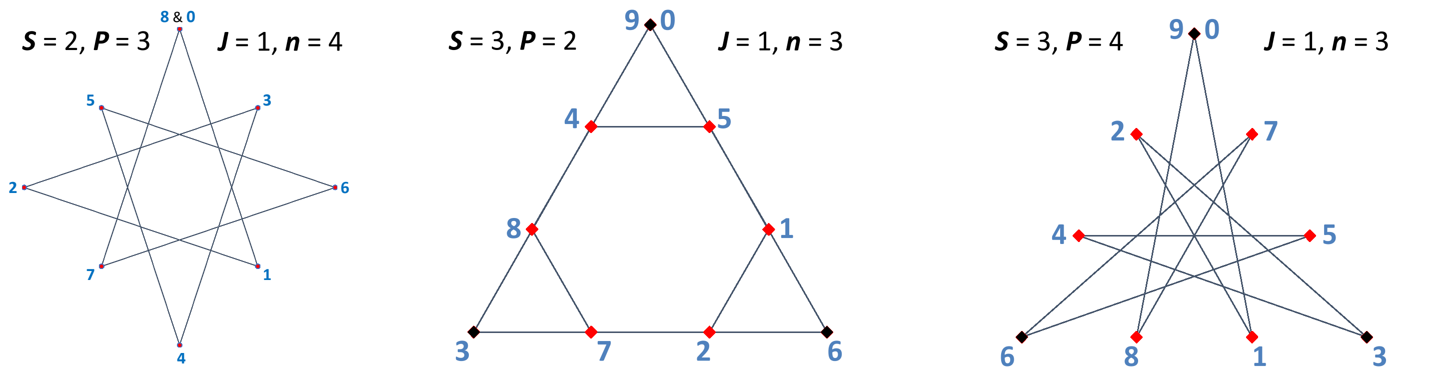
From: https://blogs.dickinson.edu/playing-with-polygons/ File 2

Finding the Total Number of Connected Line Segments in an Image

A circuit is completed once the top of the polygon (the starting point of the image) is used as a segment end point. With the annotated images used to explain the difference between S and P shown below, this occurs at the 8th segment in star on the left, or the 9th segment for the middle and right images. Both occur after $n*S$ connected segments (8 = 4*2 and 9 = 3*3) because P has no divisor in common with $n*S$.



The maximum number of possible segments is $n*S$ because that is how many subdivision endpoints exist given n vertices and S subdivisions between consecutively used vertices. Not all of these endpoints need be used, however. For example, if $P = S$, no subdivision endpoints except the polygon vertices themselves will be used, regardless of the size of S (if $P = 2$ in the left example, a square results or if $P = 3$ in the other two, a triangle results). Similar issues occur when J has a common factor with n (if $J = 2$ in the left example, the resulting image would be a vertical line since $n = 4$).

The total number of line segments used to complete a circuit and create an image depends on S, P, J, and n. Creating an image is a two-step process: 1) Setting the vertices being used, then 2) setting the subdivisions being used. Each step requires finding if there are common factors involved between two or more of the parameters used to create the image.

On the relative size of J and n: If $J = n$, the image degenerates to a point. Suppose $K > n$ where K is the number of vertex jumps. This requires that you count around the circle at least once before landing on the K^{th} vertex. That vertex is one of n possible vertices, so the same image occurs using $J = \mathrm{MOD}(K, n) < n$, where MOD is the remainder function.

On the *vertex common factor*, *VCF*: If there is no commonality between n and J, then all of the polygon's vertices will be used. If there is commonality, then fewer vertices will be used. For example, if $n = 12$, all vertices are used when J = 1, 5, 7, or 11. J = 1 and 11 produce a dodecagon and J = 5 and 7 produce 12-point stars. If J = 2 or 10, half as many vertices are used and a hexagonal image results. If J = 3 or 9, one third of the vertices are used and a square image results. If J = 4 and 8, one fourth of the vertices are used and a triangular image results. And when J = 6, a vertical line results.

Mathematically, the *vertex common factor*, ***VCF***, is: $\mathbf{VCF} = \mathbf{GCD}(J, n)$,

where **GCD** is the greatest common divisor function.

The number of vertices used, v_{used}, is then given by: $v_{used} = n/\mathbf{VCF}$.

On the *subdivision common factor*, *SCF*: On each of the line segments connecting used vertices, v_{used}, we create S subdivisions. The total number of possible subdivision endpoints is thus $S*v_{used}$. Not all of these endpoints are used if P has factors in common with $S*v_{used}$.

Mathematically, the *subdivision common factor*, ***SCF***, is: $\mathbf{SCF} = \mathbf{GCD}(P, S*v_{used})$.

The number of lines in the image, L, is then given by: $L = S*v_{used}/\mathbf{SCF}$.

An Example: Suppose $S = 8$, $P = 20$, $J = 9$, and $n = 12$. There are $n*S$ = 12*8 = 96 possible line segments in the final image. But **VCF** = GCD(9, 12) = 3, so v_{used} = 12/3 = 4. Similarly, the **SCF** = GCD(20, 8*4) = 4 so that L = 32/4 = 8. Indeed, the image is on the left, above.