## Visualizing the Vertex Frame: The visible number of line segments might differ from the calculated number of segments.

On the vertex frame. The vertex frame is the set of line segments which include all possible subdivisions. The vertex frame is solely determined by $\boldsymbol{n}$ and $\boldsymbol{J}$ so that we see that the vertex frame is either a polygon or a continuously drawn star as defined in File 1. We see the vertex frame of an image when $\boldsymbol{S}=\boldsymbol{P}$. It is worth explicitly noting that all subdivision endpoints used to create an image are points on the vertex frame.

The line segments that comprise the vertex frame need not be part of the final image. In fact, if $\boldsymbol{P}>\boldsymbol{S}$, then NO parts of the vertex frame are on the image except subdivision endpoints. Conversely, if $\mathbf{P}<\boldsymbol{S}$, then at least some part of each line segment creating the vertex frame is included in the image. In any event, it is worthwhile considering what the frame looks like because all lines in the image connect subdivision points which are themselves on the frame.

The visible number of line segments might differ from the calculated number of segments. There are a variety of reasons why this may occur. Two reasons emerge in File 2.

1) If $\boldsymbol{S}$ is a multiple of $\boldsymbol{P}, \boldsymbol{S}=\boldsymbol{m}^{*} \boldsymbol{P}$, then each line in the vertex frame is created by connecting $m$ colinear line segments (colinear means on the same line).

Take the example to the right with values of $\boldsymbol{n}, \boldsymbol{J}$, and $\boldsymbol{S}$ listed. Four of the six values of $\boldsymbol{P} \leq 6$ produce the vertex frame (because $\boldsymbol{S}=6$ ) which has three apparent line segments in the final image (since $\boldsymbol{n}=3$ ). But the calculated number of line segments (shown in the Lines column) vary dramatically because many of these segments are colinear parts of the triangular vertex frame.

Each line in the vertex frame contains $\boldsymbol{m}$ segments in this
 instance so if $\boldsymbol{P}=1, \boldsymbol{m}=6$, and so on for other values of $\boldsymbol{P}$.

2) The triangular image at the left, used to explain $\boldsymbol{S}$ versus $\boldsymbol{P}$ counts 9 line segments to create the image.

Visual inspection of this image leads to the impression that there are only 6 segments: the three lines in the vertex frame and the three lines from 1 to 2,4 to 5 , and 7 to 8 .

The difficulty here is that, in order to complete the image while maintaining the counting pattern (i.e., to be continuously drawn), one needs 9 line segments to create the 6 line final image. Some of those segments partially overlap in achieving this image. For example, the last half of the $1^{\text {st }}$ segment coincides with the first half of the $6^{\text {th }}$ segment.
3) Additional reasons why the visible number of line segments differs from the calculated number of line segments are found in later files.

