

Shape-shifting Polygons

When n was just-under or just-over a multiple of J we saw Rotating Polygons in 1.4. These polygons maintained their shape as they rotated about the vertices of the parent polygon.

After incorporating subdivisions and points, we obtain similar results when the number of lines $L = S*n$ (assuming VCF and SCF are both 1), is just-under or just-over a multiple of P . We can visualize the problem in the opposite direction: for fixed S and n , find P that is close to a common fraction of $S*n$. By working with a prime polygon such as $n = 19$, we can eliminate $VCF > 1$ issues. Using $S = 21$ provides a nice solution because $S*n = 399$. (As an aside, note that this is the difference between two squares, 20 and 1, $21*19 = (20+1)*(20-1) = 20^2 - 1^2 = 400 - 1 = 399$.) In this instance, we will need to adjust P so that P is not divisible by 3 or 7. (As a second aside, had we used $29*31$ we would have had no such issues since both are prime. The difficulty here is that the images have more than twice as many lines (899) and are a bit too dense to copy and paste.)

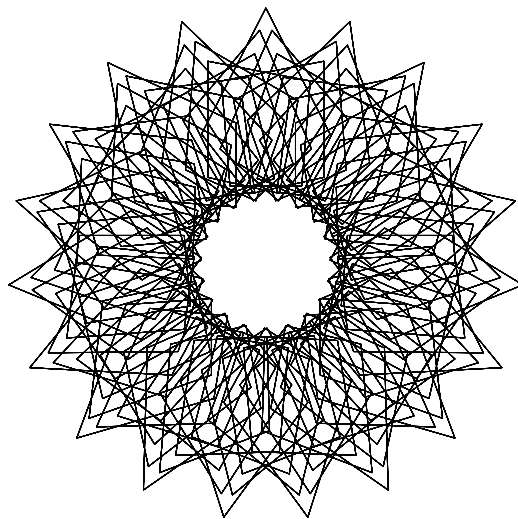
Consider $P = 100$ which leads to $S*n$ being just under $4P$. Quadrangles should result. And indeed, they do. The square in the $J = 1$ version can be seen here once you click *Toggle Drawing* (as will be the case for the rest of these images):

<https://www.playingwithpolygons.com?vertex=19&subdivisions=21&points=100&jumps=1> looks almost the same as its rotating polygon cousin from 1.4. The only difference is that there is a bit of “wobble” in the square (that is not visible) because only 19 of the star’s 399 vertices are polygonal vertices, the rest are on the line segments between polygonal vertices. This produces a faint 19-point star. Compare to the $n = 399$, $J = 100$, rotating polygon image in which square rotates smoothly 100 times: <https://www.playingwithpolygons.com?vertex=399&subdivisions=1&points=1&jumps=100>.

The “square” in the $J = 3$ version bounces around a bit more noticeably, but still maintains the appearance of having equal sides and right angles: <https://www.playingwithpolygons.com?vertex=19&subdivisions=21&points=100&jumps=3>.

However, the quadrilateral in the $J = 5$ version begins to show the *shape-shifting* attribute induced by S and P . To see this increase J from 3 to 5 in the above link. Some sides are clearly longer than others and the angles are obviously not right angles anymore. This same thing happens for $J = 7$ which, from a stylistic drawing perspective, is similar to $J = 5$ (just as $J = 1$ and $J = 3$ are similar).

The most interesting quadrangle versions may well be $J = 8$ and 9. The $J = 8$ version, shown to the right, maintains the convex quadrangle images like $J = 5$ and 7, but now the quadrangles are smaller, and they rotate around the perimeter excluding the center. Each 21 line cycle includes 5.25 quadrangles ($21 = 5*4+1$). Each cycle spans two polygonal vertices so this is drawn as a *twice-around image*. The 9 cycles in the “1st-time-around” end in even vertices and the 10 cycles in the “2nd-time-around” end in odd vertices. Click *Toggle Drawing* to see this cyclical behavior: <https://www.playingwithpolygons.com?vertex=19&subdivisions=21&points=100&jumps=8>.



The quadrangles in the $J = 9$ version are no longer convex; they now resemble arrowheads. The outward pointing peak is reasonably fixed in size but the base vertex (which creates the nonconvexity) undulates from side to side. Click *Toggle Drawing* to see this happen: <https://www.playingwithpolygons.com?vertex=19&subdivisions=21&points=100&jumps=9>.

We have thus far ignored the $J = 2, 4$, and 6 images. Each has a 21 segment cycle, just like other $S = 21$ images. $J = 2$ and $J = 6$ both involve successive line segment movements across the center of the image which makes following the individual jump structure hard to see ($J = 2$ appears similar to a Porcupine Polygon but it is not). By contrast, the five quadrangles in each $J = 4$ cycle are very easy to see, they are like the lights on a string of Christmas lights.

<https://www.playingwithpolygons.com?vertex=19&subdivisions=21&points=100&jumps=4>. This link has a very nice shape-shifting image: <https://www.playingwithpolygons.com?vertex=30&subdivisions=29&points=217&jumps=13>

Exercise: Construct triangular shape-shifting polygons for $S = 21$ and $n = 19$. What is P and what happens as J changes?