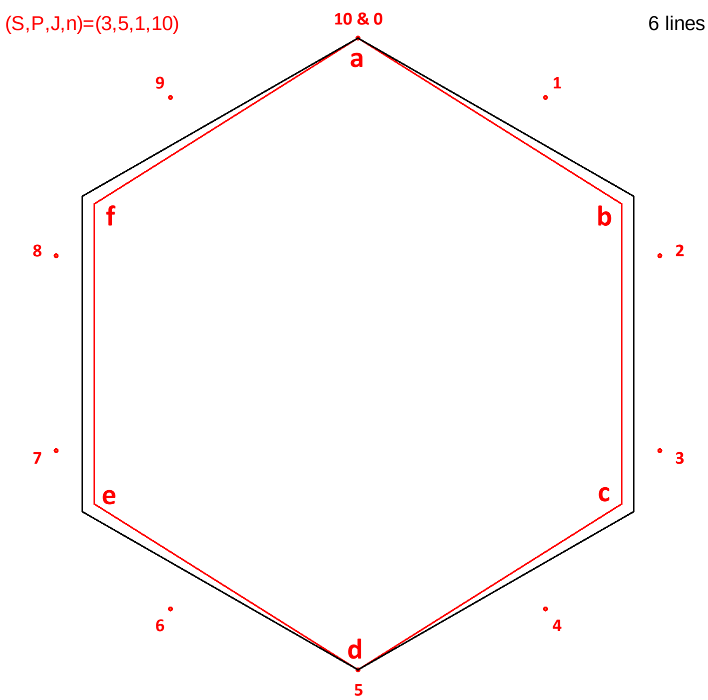
Lines of Symmetry, Take 2

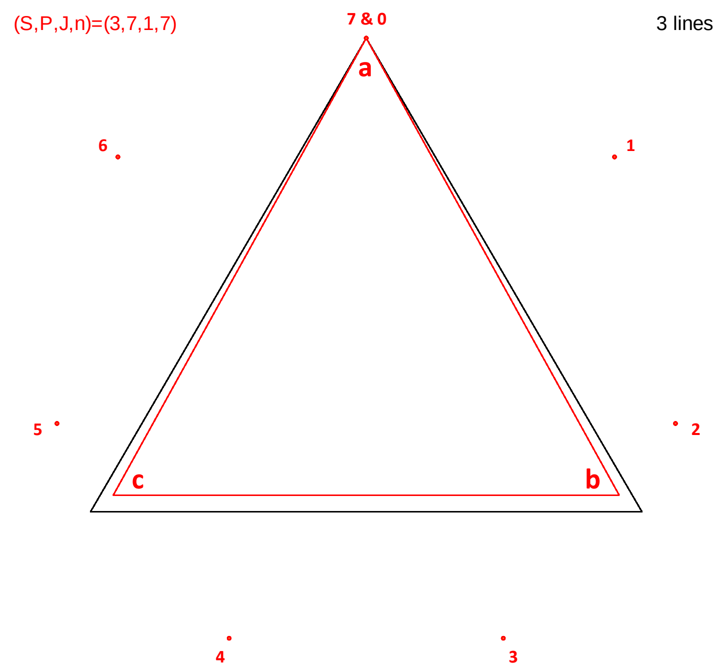
Both images below were created by superimposing a **black regular polygon** on a **red irregular polygon** (black polygonal vertices are suppressed to avoid confusion). Regular polygons have equal sides and angles (the black hexagon angles are 120° and the black triangle angles are 60°). Both red polygons are irregular because the angles at each vertex are not all the same. Visual observation confirms that the red's angle at **a** in both images is smaller than their regular counterparts. Red's angle at **d** in the hexagon is identical to **a** (each is constructed 2/3 of the way along the second line from the vertex point). By contrast, red's hexagonal angles at the other four vertices (**b**, **c**, **e**, and **f**) are the same as one another and are larger than 120° and red's triangular angles at **b** and **c** are the same as one another and are larger than 60°.

Claim: *A line of symmetry (***LOS***) maps lines to lines and angles to angles.* This leads to the following conclusions.

Black. 6 **LOS**: 3 are vertex to vertex, 3 are midpoint to midpoint. 3 **LOS**: One each from vertex to opposing midpoint.

Red. 2 **LOS**: 1 vertical, **a** to **d**; 1 horizontal, **2.5** to **7.5**. 1 **LOS** from **a** to the midpoint of segment **bc** (point **3.5**).
(In the **Red** hexagonal image, **be** is not a **LOS** because **a** ≠ **c** and midpoint **ab** to midpoint **de** is not a **LOS** because **a** ≠ **b**.)





The red points used above (**2.5**, **3.5**, **7.5**) are midpoints between vertices. These midpoints allow us to create lines of symmetry without resorting to doubling ***J*** and ***n***. Lines of symmetry may exist between the following pairs of points:

(a) vertex ***k*** and ***n***/2 + ***k*** and (b) ***k*** + 1/2 to (***n***+1)/2 + ***k*** for 0 ≤ ***k*** < ***n***/2.

If ***n*** is odd, the second point in (a) is a midpoint between vertices but the first point in (b) is a midpoint.

If ***n*** is even, both points in (a) are vertices, and both points in (b) are midpoints between vertices.

These lines may not be lines of symmetry, but all lines of symmetry must satisfy one of these two conditions.

The image at right has 4 lines of symmetry: **0** to **2**; **0.5** to **2.5**; **1** to **3**; and **1.5** to **3.5**. The 1^{st} and 3^{rd} are (a) and the 2^{nd} and 4^{th} are (b).