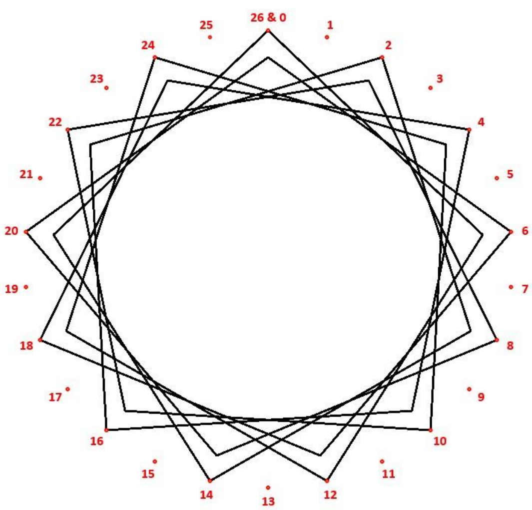
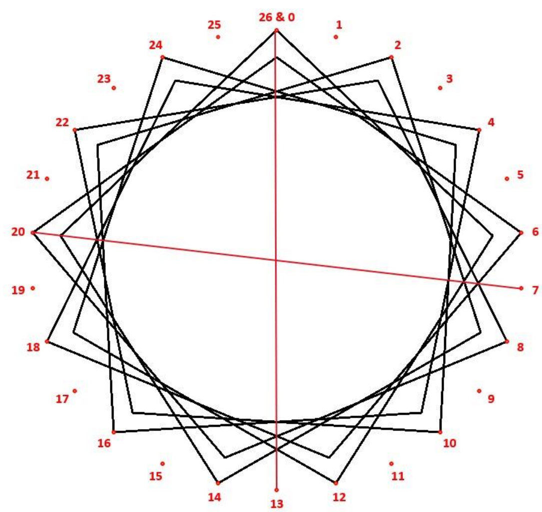
Lines of Symmetry

An image has **symmetry** if its parts have a property of "sameness." Two forms of sameness are reflective and rotational. Here we examine reflective (or linear) symmetry. If a line can be drawn across an image that breaks the image into two mirrored parts, this line is called a **line of symmetry** (**LOS**). If you click *Show vertices and labels* on the MAIN page of *File 2,* you can examine lines of symmetry drawn between vertices. This allows you to see all the vertices enclosing your image numbered from 1 to ***n***.

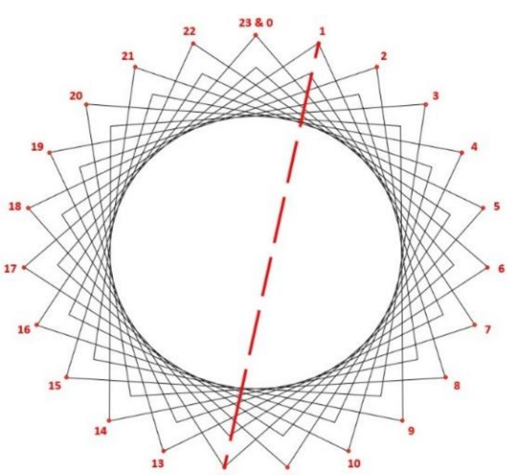
For the ***S***, ***P***, ***J***, ***n*** = 2, 3, 4, 26 images below, lines of symmetry can be easily drawn between some vertices, but not all. One interesting **LOS** can be noticed quickly, the line from vertex **26&0** to vertex **13**, where one vertex is attached to the image while the other is not. This may be the most easily recognizable line of symmetry since it is created with a vertical line. However, other less noticeable lines of symmetry can also be observed, like the line between vertex **20** and vertex **7**, which is slightly tilted. As we can see, vertices that act as endpoints of a **LOS** need not look similar, nor be attached to the image, to have a line of symmetry drawn between them.



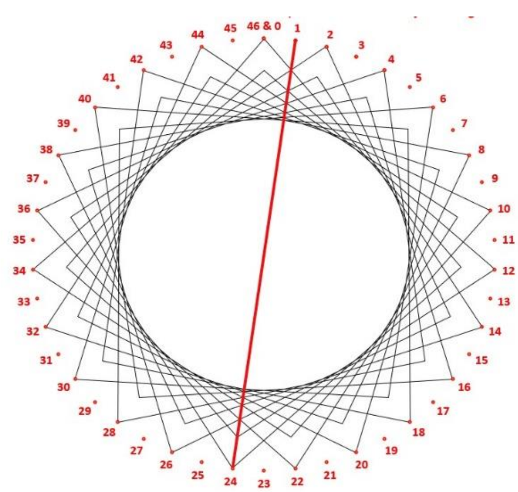


In the images above, a line of symmetry may only be drawn between two vertices k and $\frac{n}{2}+k$, where k is any integer less than **n**/2.

We may be tempted to think that this k and $\frac{n}{2}+k$ pattern can draw a **LOS** for any polygon, but this is not the case! For example, the pattern will never occur when ***n*** is odd. This is because, if ***n*** is odd, then ***n***/2 is not an integer, and thus there is no vertex at $\frac{n}{2}+k$. For example, in the ***S***, ***P***, ***J***, ***n*** = 2, 3, 4, 23 bottom left image, we cannot draw a **LOS** between vertices. Although the line from vertex **1** to vertex **12** may at first look like a line of symmetry, it is not, it does not pass through the center of the image which all lines of symmetry must do. While lines of symmetry do exist for this image, none can be drawn between vertices on the left.

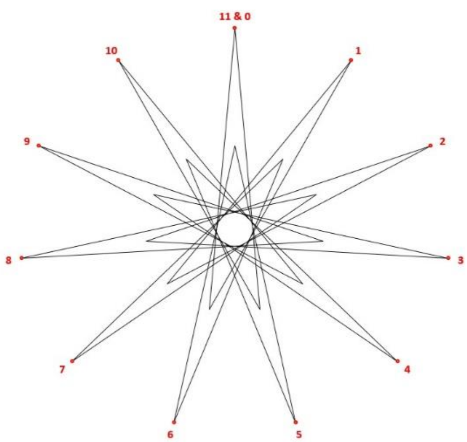


To create an image identical to the 2, 3, 4, 23 image but containing vertex lines of symmetry, double ***J*** and ***n*** to produce the right image.

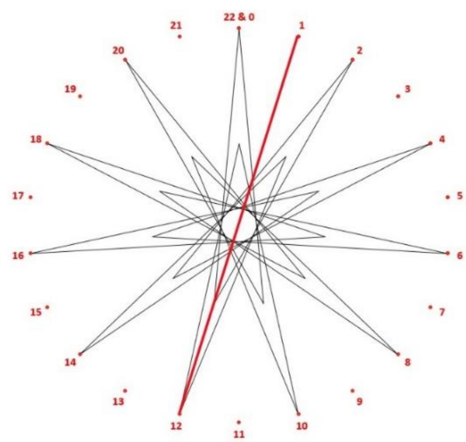


(This has no effect on the image produced, the only change is that now **VCF** = 2.) Now the line drawn from **1** to **24** is a line of symmetry. This line follows the k and $\frac{n}{2}+k$ pattern given $k = 1$ and $n = 46$.

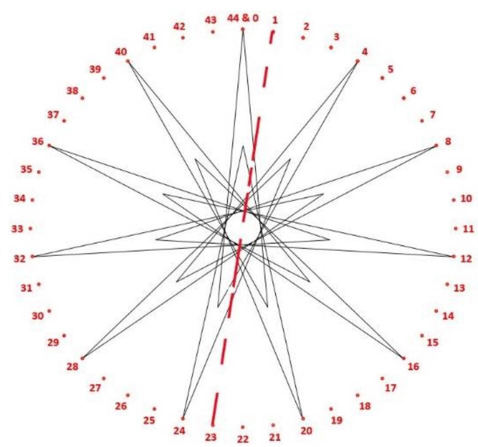
We must be careful to note that the k and $\frac{n}{2}+k$ pattern does not simply work anytime n is even. For example, consider the 3 images below. These are all *Star in a Star* images. The leftmost image (2, 3, 4, 11) cannot have lines of symmetry between polygonal vertices because n is odd. However, when we double n and J, observed in the middle image, we can now draw **LOS** between any two vertices k and $\frac{n}{2}+k$. However, say we double n and J once more from here to create the 2, 3, 16, 44 image, shown at right. Although there are many lines of symmetry that can be drawn here and all follow the k and $\frac{n}{2}+k$ pattern (for example, vertices **2** and **24**), the k and $\frac{n}{2}+k$ pattern does not work for all k in this image. For example, if we set $k = 1$, we may be tempted to think that we could draw a **LOS** between vertices **1** and **23**. However, this dotted red line on the right is certainly not a line of symmetry, despite passing through the center of the circle and being a point that satisfies this pattern.



2, 3, 4, 11



2, 3, 8, 22



2, 3, 16, 44

With all of this in mind, we can say four things about drawing lines of symmetry in *File 2.*

1. To even be able to draw lines of symmetry between vertices, n must be even.
2. Any **LOS** that we can draw must be drawn between two vertices k and $\frac{n}{2}+k$.
3. Even when n is even, and we can draw a line between k and $\frac{n}{2}+k$, this may still not be a **LOS**.
4. We can, however, guarantee that one **LOS** will always exist in any image (although for odd n it cannot be drawn between vertices) and this is the line that continues from the **n&0** vertex through the center of the image. In other words, the center vertical line is always an **LOS**.

Finding symmetry requires a careful eye and close analysis by the observer; we cannot just draw lines across a star and expect it to be a line of symmetry! When attempting to draw a line of symmetry, we must make sure to always ask ourselves if the line acts as a mirror across our image.