## Using Cycles to Understand Images

The Extra Materials For Instructors includes a version of File 2 that allows you to create materials suitable for classroom presentation, but the file can also be used to help understand the underlying aspects of a given image.

One toggle in particular helps to show how the image was created by letting the user scroll through the first $\boldsymbol{k}$ lines in the image (cell B10). The value of $\boldsymbol{k}$ can be set by typing a number in cell C11, using the up/down arrow keys in C10:12, or by linking C11 to a specific cell.

A useful cell to link $\boldsymbol{k}$ to is $\boldsymbol{S}$ (type $=\mathrm{C} 1$ in cell C11) because this shows the first cycle as long as $\boldsymbol{S}$ and $\boldsymbol{P}$ have no factors in common. (If the Equations toggle in cell C6 is clicked on, then you could alternatively type $=$ M9 in C11.)

The image to the right shows the first cycle, the vertex frame, and labels. The first cycle ends at vertex 1 and has 38 lines. The endpoints of these 38 lines are subdivision points on 9 of the 20 lines on the vertex frame.


The used vertex frame lines can be seen as the lines from: 12-1, 13-2, 14-3, 15-4, 16-5, 17-6, 18-7, 19-8, and 20\&0-9.
The image below shows how 20 such cycles create the full image, one cycle spanning each pair of successive vertices.

$$
(\mathrm{n}, \mathrm{~S}, \mathrm{P}, \mathrm{~J})=(20,38,169,9) \quad 760 \text { lines }
$$



Click here then click Toggle Drawing to see this drawn. It is worth noting a couple of things about nearby images.

1. If you change $\boldsymbol{S}$ with fixed $\boldsymbol{n}, \boldsymbol{P}, \boldsymbol{J}$ and $\operatorname{GCD}(\boldsymbol{S}, \boldsymbol{P})=1$ then the first cycle will continue to end at $\mathbf{1}$; the image remains a one-time-around image. $\boldsymbol{S}=34$ looks like a pulsing square and $\boldsymbol{S}=31$ is a 20-point spinning needle star.
2. If you change $\boldsymbol{P}$ for fixed $\boldsymbol{n}, \boldsymbol{S}$, and $\boldsymbol{J}$ the images produced are no longer necessarily one-time-around. For example, $P=167$ looks very similar to $\boldsymbol{P}=169$ but it is a 3 time around image because the first cycle ends at vertex 3 .
