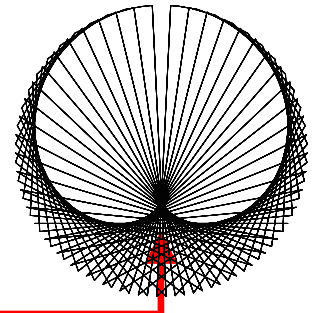


Cardioid Basics

Cardioids can be created in a number of ways (as a quick perusal of Wikipedia will confirm), but the method employed here is to use a series of straight lines (starting at each of the polygon's n vertices) to create the image.

If the cardioid is based on a n -gon (like the $n = 99$ version shown to the right), then a maximum of $n-1$ lines are used to create the final image. (This is a maximum because some lines may overlap, and some "lines" may start and end at the same point.) Unlike the images in **PART I**, lines are drawn one at a time and need not be connected to one another. When n is large enough, the individual lines appear to create a symmetric image with a single **cusps** or gathering point.



Creating a classic cardioid. The rule for drawing lines is simple: Consider each vertex j from 1 to $n-1$ to be a starting point of a line segment. The other end of that line segment is at vertex $2j$. Call this line $(j, 2j)$.

When $j > n/2$ then $2j > n$, and the end point is $2j - n$. More generally, the end point is the remainder once $2j$ is divided by n . NOTE: There is no need to check for n as a starting point because if the starting point is n , the ending point, $2n$, is the same as n once we subtract n .

The image to the right shows the 9 segments created by following this rule given $n = 10$. This value of n is particularly easy to work with because one subtracts 10 from the endpoint by simply ignoring what is in the 10's place in the final number. The 9 lines are:

$(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 2), (7, 4), (8, 6), (9, 8)$.

Notice the vertical symmetry in the resulting image.

Extending the basic model. We obtain equally interesting images if we replace the final end point multiplication factor, k , by values of k other than 2.

The image to the right shows the 8 lines created given when $n = 10$ and $k = 3$. These lines are:

$(1, 3), (2, 6), (3, 9), (4, 2), (6, 8), (7, 1), (8, 4), (9, 7)$.

It is worth noting a couple of points in this instance:

- 7 connects with 1 because 1 is the remainder of 21 ($21 = 3 \cdot 7$) once 21 is divided by 10. A similar statement could be made about the lines starting at 8 and 9.
- There is no line starting at 5 because that starting point coincides with its ending point. The remainder of 15 ($15 = 3 \cdot 5$) upon division by 10 is 5.
- If you examine larger values of n given $k = 3$ you will see that now there are two cusps rather than one. What happens when $k = 4$, or 5, or ...? Use the file to find out. The file allows $n \leq 360$ and $k \leq 360$.

