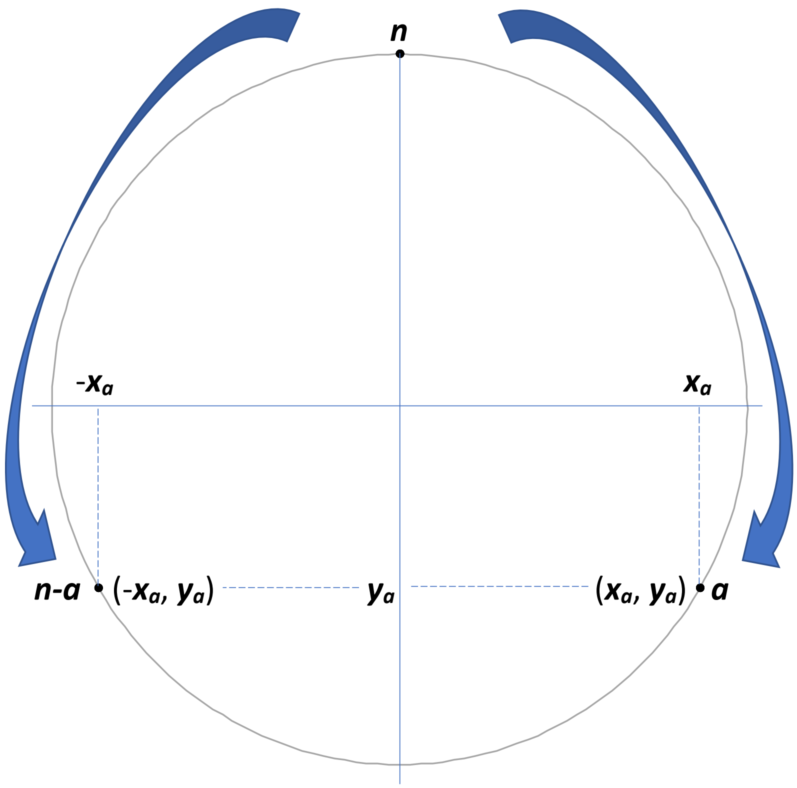
Why Cardioids have Vertical Symmetry

Cardioid images are symmetric for any ***n*** and ***k*** about the diameter going through ***n***, the top vertex. One way to show this symmetry is to consider paired individual vertex points and lines created using those starting points.

It is worth remembering that by construction, vertex ***j*** and vertex ***n-j*** are at the same height (or ***y*** coordinate in the (***x***, ***y***) plane). This was one of the key points used to make triangular images with horizontal lines in **PART II**.

Symmetric Points. The image to the right allows us to consider how the ***x*** coordinates of vertex ***a*** and ***n-a*** compare to one another. The first is ***a*** vertices from the top vertex, counted clockwise. Let the (***x***, ***y***) coordinate of this point be (x_a, y_a). The second is ***a*** vertices from the top, counted counter-clockwise. The vertex ***n-a*** has the same ***y*** coordinate but the opposite ***x*** coordinate from vertex ***a***, or ($-x_a$, y_a). These are mirror image points using the vertical diameter (through point ***n*** and described by ***x*** = 0) as the line of reflection. We can see the symmetry involved here by simply considering whether vertices are being counted clockwise or counter-clockwise around the circle.



Symmetric Lines. Lines created from symmetric paired starting points are themselves symmetric. It is worth considering two pairs of starting points, 1 and ***n***-1 and 2 and ***n***-2 before examining a general starting point $j < n/2$.

The right-hand side of the table is our typical starting point because we count vertices around the circle clockwise starting at the top. Vertex 1 connects to ***k*** and 2 connects to 2***k***. Compare those lines to the lines starting at ***n***-1 and ***n***-2. Reorganizing the ending point by removing the factor (***k***-1)***n*** has no effect on the location of the ending point because all that matters is the remainder upon division by ***n***. **The red bracketed term** for each ending point on the left-hand side is related to the right-hand side ending point in exactly the same way as starting points as well as the image showing ***a*** and ***n-a*** above. Since starting and ending points are symmetric to one another, so is the line segment connecting those points.

The same relation exists for a general starting points ***j*** and ***n-j*** and ending points ***jk*** and (***n-j***)***k*** by the same reasoning. Both share the same ***y*** coordinate, but the ***x*** coordinates of the two points are the same magnitude but have opposing signs. And once again, since starting and ending points are symmetric to one another, so is the line segment connecting those points.

Location of the Starting Vertex

Left-hand side		Right-hand side	
Starting Point	Ending Point	Starting Point	Ending Point
$n-1$	$k(n-1) = kn - k = (k-1)n + [\,n-k\,]$	1	k
$n-2$	$k(n-2) = kn - 2k = (k-1)n + [\,n-2k\,]$	2	$2k$
$n-j$	$k(n-j) = kn - jk = (k-1)n + [\,n-jk\,]$	j	jk