Measuring the Angles created by Drawing Lines between Regular Polygonal Vertices

You can readily determine the size of an angles created when two lines connecting polygon vertices intersect one another. If this occurs at a polygonal vertex, the angle is called an *inscribed angle*, otherwise it is an *interior angle*. In the image below, inscribed angles occur at three of the vertices, 9, 3 and 6 (vertices below are denoted by the black numbers 1 to 9 given the 9-gon shown). The other five intersections create interior angles. Both can be described in two ways: first using *degrees* (in **red**); but more easily by *the number of vertices spanned by their arms* (shown in **blue**). (Arms are the sides of an angle so that the inscribed angle denoted 9-3-1 has arms 9-3 and 1-3 with vertex at 3.)

Smallest Central Angle: The arc between adjacent vertices of a regular polygon has a *central angle* (angle whose vertex is the center of the circle) of $(360/n)°$ ($40°$ in the 9-gon below). If there are a arcs, then the central angle is $(360a/n)°$.

Inscribed Angle: The *inscribed angle theorem* says that an inscribed angle is half size of its central angle or $(180a/n)°$. In the image below, angles that span a single arc (like 9-3-1 and 8-6-9) therefore have an inscribed angle of $(180/9)°$ = **20°** (**red**) or span one **1** arc (**blue**). Similarly, 6-9-3 is **60°** or **3** arcs (from 3 to 6) and 9-3-7 is **40°** or **2** arcs (from 7 to 9).

Interior Angle: Interior angles are measured using the sum of arcs on both sides. For example, the interior angle created between 1 and 2 from lines 1-3 and 2-6 has **4** arcs (or **80°**), one arc between vertices 1 to 2, and three arcs between vertices 3 and 6. The supplementary angle to this angle has **5** arcs (or **100°**), one between 2 and 3 and the other four between 6 and 1. Note that supplementary angles sum to **180°** because the sum of arcs for both angles is always n.

General equations for angle measures are provided in the Cardioid file but follow the rules laid out here.

9 n polygon vertices   **3** k, Multiplier