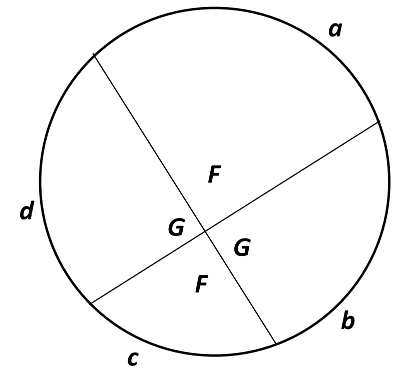
About Right Angles in Cardioids

All angles created by the intersection of two lines must be acute (less than 90°), right (90°), or obtuse (greater than 90°). While acute and obtuse angles occur in the vast majority of ***n***, ***k*** combinations, for a right angle to exist, one thing is certain: ***n*** must be even.



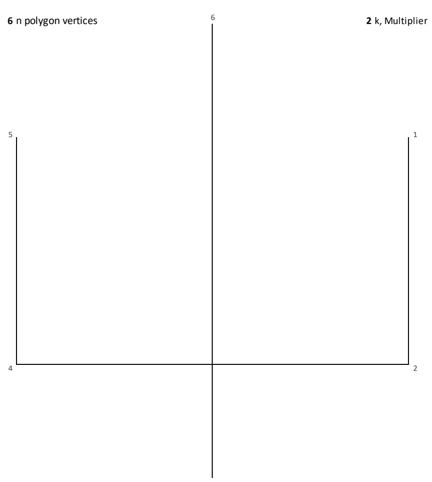
Why *n* is even. Consider the diagram to the right for an interior angle. Let ***a***, ***b***, ***c***, and ***d*** represent non-overlapping arcs of a circle that sum to ***n***. Lines connecting alternating arc endpoints create two angles ***F*** and ***G***. These angles must satisfy

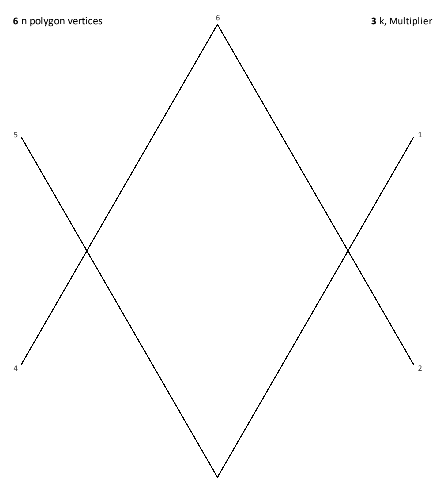
$$F = 180(a + c)/n^{\circ} \qquad G = 180(b + d)/n^{\circ}$$

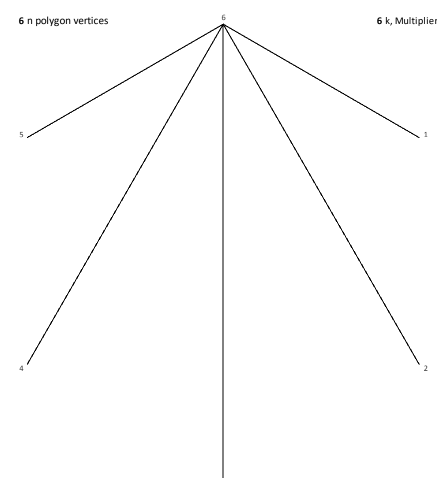
These angles are right if ***F*** = ***G***. This implies (***a*** + ***c***) = (***b*** + ***d***). Since (***a*** + ***b*** + ***c*** + ***d***) = ***n***, 2(***a*** + ***c***) = ***n*** or (***a*** + ***c***) = ***n***/2. Since ***a*** and ***c*** are whole numbers, ***n*** must be even.

Inscribed angles can only be right angles if ***n*** is even because the arc contained by the arms of the angle, ***a***, must be ***n***/2 given 90° = 180·***a***/***n***° according to the *Inscribed Angle Theorem*. It is worth noting that if an inscribed angle is right, then if one were to draw a line connecting the endpoints of the arms, the resulting line would be a diameter of the circle (with half the vertices of the polygon on each side of this line). We will see this in the discussion of right angles at vertices 2, 4, and 6 in the ***n*** = 14, ***k*** = 2 bottom image.

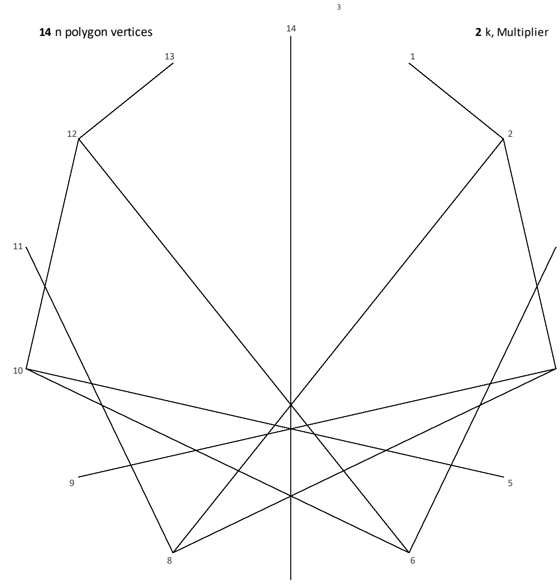
It is instructive to note that an even ***n*** does not ensure that the image has right angles because ***k*** might produce an image without right angles. The three examples below are ***n*** = 6 with ***k*** = 2, 3, and 6. The middle has no right angles while the left has two inscribe and one interior right angle vertex point and the right has two inscribed right angles.







The ***n*** = 14, ***k*** = 2 example at the right has 6 inscribed right angles, one at each even vertex except for ***n*** = 14. Recalling that cardioids have vertical symmetry, it is worth considering why right angles exist at vertices 2, 4, and 6. The reason is based on the starting point vertices for each arm of each angle. For each point, remember that ***n***/2 = 7 and ***k*** = 2 so the end point is always 2 times the starting point and when the endpoint is larger than ***n***, all that matters is the remainder.



- ⊾2) 1 to 2 is one arm. The other arm is from ***n***/2+1 = 8 to ***n*** + 2 or 2.
- ⊾4) 2 to 4 is one arm. The other arm is from ***n***/2+2 = 9 to ***n*** + 4 or 4.
- ⊾6) 3 to 6 is one arm. The other arm is from ***n***/2+3 = 10 to ***n*** + 6 or 6.

In each instance, the starting point of each arm is half-way around the circle from the other starting point, just as required for right angles.