The Curved Image is Created 180(k+1)/n° per Line

The cardioid below can be visualized as being composed of two parts, the "3/4-of-a-turn spiral" which reverses itself upon reaching the vertical midline to create an inverted heart, and the spray of lines that seemingly emerges from behind the heart as a kind of fan like structure. Both parts are, of course, linked to one another.

The n = 54, k = 2 cardioid image below is created from 52 **distinct** lines, one line *starting* at each vertex except for vertices 36 and 54. The vertical line ending at 54 started at 27 and the horizontal line starting at 36 ends at 18 because 72 = 2·36, and 72 = 54+18 but this line coincides with the line starting at 18 and ending at 36 = 2·18.

We focus on vertices 1 to 27 here due to the symmetry involved in cardioid images. These vertices create the right half of the inverted heart and the left half of the fan. We consider the lines from 1 and 2 differently from the rest of the lines.

The line from 1 to 2 has a slope of 10°. Imagine the angle 1-2-52. It has an angle of 180·3/54 = 180/18 = 10° according to the Inscribed Angle Theorem because the arc between 52 and 1 is three vertices. But the 2-52 arm of the angle is horizontal so that the line starting at 1 has a slope of 10°. By similar reasoning, the line from 2 to 4 has a slope of 20° because the angle 2-4-50 has an arc of six vertices between 50 and 2 or slope of 20° and the 4-50 arm is horizontal.

The successive starting point interior angle is always 10°. From here on, we consider the interior angle created by each pair of successive starting point vertices. The first such angle is the one created by the intersection of the 2 to 4 line and the 3 to 6 line. This angle is also 10° because of an arc of 1 on the starting side (from 2 to 3) and 2 on the ending side (from 4 to 6). Put another way, the line starting at 3 has slope of 30°. This pattern continues with each successive vertex.

Note in particular that the line starting at 9 has slope of 90° or is vertical, and the line starting at 18 is horizontal (180°) and the line starting at 27 is once again vertical (270°). The curve creating the right half of the inverted heart is the envelope of 27 line segments turning 10° at a time in going half-way around the 54 starting vertices.

The fan of lines ending at even vertices from 38 to 54 are simply the continuation of starting vertices from 19 to 27.

The same patterns emerge when k > 2. One needs to increase n to see the curves created but these images are created using successive starting point vertices turning 10°: n = 72, k = 3; n = 90, k = 4

Note that in both cases, the line starting at 9 (and n-9) is vertical just like this image.

Of course, there is nothing special about 10° (other than it is easy to keep track of degrees by add a zero to the starting vertex). For example, if you set n = 96, k = 5, individual lines will increase by 180·(5+1)/96 = 11.25° per vertex so that the line starting at 8 (11.25·8 = 90).