

Using Functional Relationships to Find Patterns

Image archetypes are easy to find using the cardioid file. The simplest of these archetypes is, of course, the classic images that occur when k is fixed at a specific value such as 2, or 3, or 13. These values of k produce 1, or 2, or 12 cusps equally spaced around the circle as noted in *explainer 11.1* once n is “large enough.”

Here we wish to examine what happens when n and k are linked to one another rather than what happens when k is constant. You can enter both numbers that satisfy the condition you wish to examine using the up/down arrow keys, but *a more effective approach is to use an equation relating the two values.*

Why we restrict ourselves to $k \leq n$. We want n and k to be whole numbers. If $k > n$ we can find a multiple m and remainder $r < n$ where $k = m \cdot n + r$ and m is a positive whole number. The remainder r once k is divided by n is described by the MOD function (first discussed in *explainer 2.1b*), so that $r = \text{MOD}(k, n)$. (For example, $k = 47$ and $n = 15$ has $m = 3$ and $r = 2$.) Consider the line starting at vertex j for two values of k : k , and r .

n, k : The ending vertex is $j \cdot k = j \cdot (m \cdot n + r) = j \cdot (m \cdot n) + j \cdot r = (j \cdot m) \cdot n + j \cdot r$.

n, r : The ending vertex is $j \cdot r$.

The j end-point is the same vertex in both instances because multiples of n have no effect on end vertex placement. This is true for all starting points; therefore, the two images are identical.

About functions. Suppose we imagine n as a function of k . For example, what happens when $n = k$? A quick check shows that this always produces a fan with $n-1$ lines. Think about how this fan was made using the cardioid rule. Start at vertex j and multiply by k . This is always a multiple of n (since $k = n$), so each line STARTS at vertex j from 1 to $n-1$ but ALL lines end at n , the top vertex. The top image is $n = k = 15$ (and each angle is 12°).

Next, consider images where $n = 2k$. You could start at $n = 4$ and $k = 2$ and increase n by 2 and k by 1 using the arrows in A1:A2 and C1:C2 and see what happens. The middle image shows one such example given $n = 30$ and $k = 15$.

Using equations. An easier way to explore a functional relation is to tie n to k and then use the up/down arrows for k because changing k will change both k AND n . The way to do this is to click on cell B1 and type **=2*D1**. [To write an equation in Excel, click where you want to enter the equation then type the equation (noted in **bold underscore**) and click enter.] As you change values of k using the arrows in C1:C2, both k and n change. You can quickly scroll through examples of $n = 2k$ using the k arrows. You will quickly notice that the image is always two fans, one at the top, the other at the bottom of the circle.

Exploring patterns in the resulting images. Stick with the $n = 2k$ image. You can quickly explore two topics 1) Are the angles radiating from top and bottom always the same (and if so, what is their size)? 2) Is there horizontal symmetry? Both topics are discussed in other *explainers* but the answer to both questions keys off of whether k is even or odd.

Once you set up a relation of n as a function of k in cell B1 it will remain in place as long as you do not use the up/down arrows controlling n in A1:A2.

Different equations produce different image archetypes. If we add 1 to the above equation so that, $n = 2k+1$, we get strikingly different images. This is easily explored by typing **=2*D1+1** in cell B1. The example to the right is the result given $k = 15$, the only difference from the middle image is that $n = 31$.

