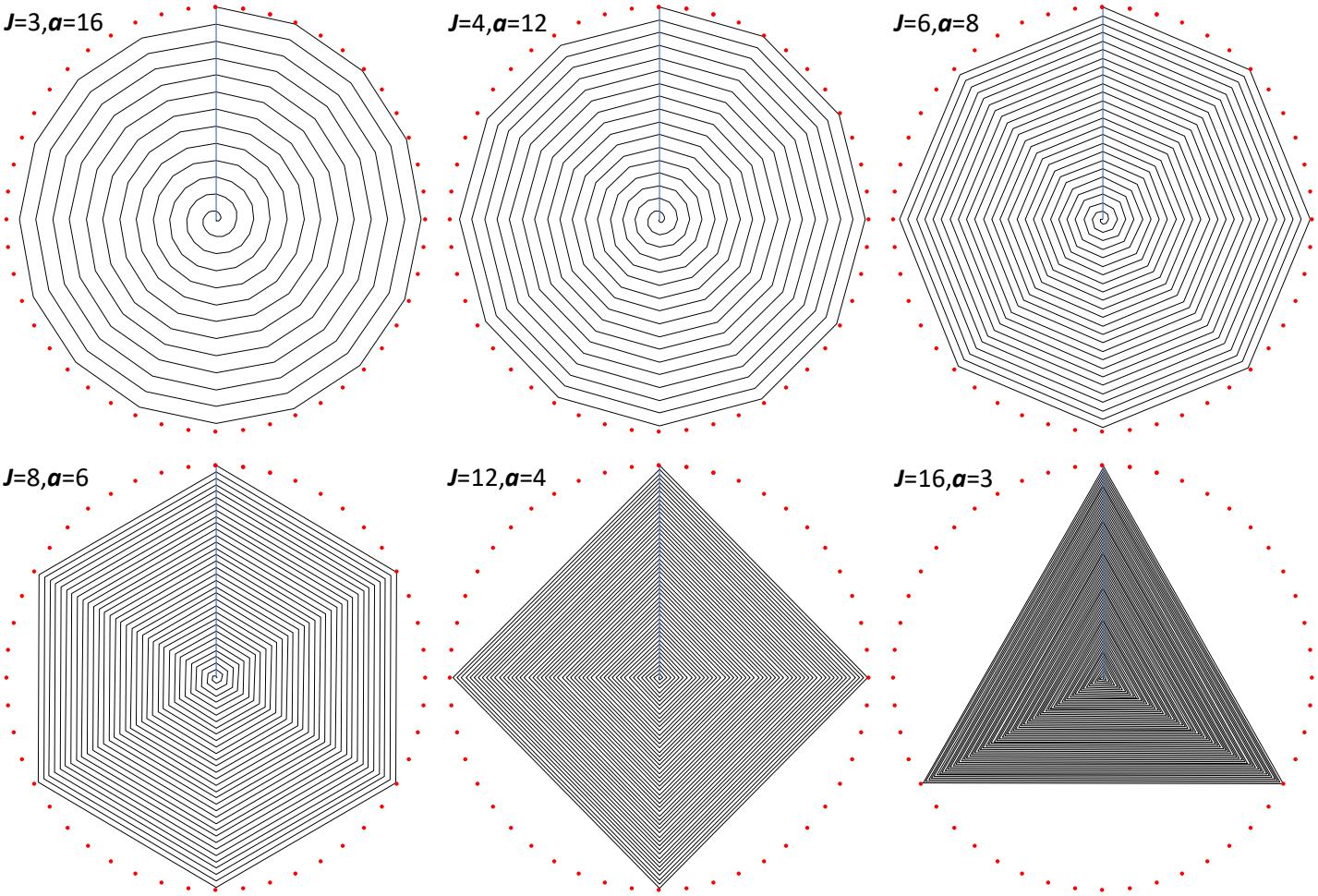
10. Almost Polygons

If n is a multiple of $J > 1$, it can be written in the form $n = \alpha \cdot J$ where α is a whole number ($J = 1$ can be used but once n is sufficiently large, the lines start to look like curves). When r is large enough the image will resemble an α sided polygon. Put differently, if n is a composite number then there will be factors $f_1, \ldots, f_k$ other than 1 and n. So, $n = 48$ has eight such factors: 2, 3, 4, 6, 8, 12, 16, and 24, the middle six of which are shown below based on $n = 48$ and $r = 200$. Each panel includes the 48 red parent polygon vertices and a blue vertical radius connecting the start of the spiral with its finish.



Each image can be seen as a polygon, or more accurately a bunch of similar polygons nested inside one another, but that is not quite true. It is *almost* a polygon. The difference is most easily seen by focusing on the blue vertical radius.

Consider the middle figure in the top row. This is NOT a dodecagon (12-sided polygon) because the endpoint of the 12th line segment is 12/200 = 6% of the way from the top to the center along the blue line – it does not connect to the top. The same can be said of the second dodecagon (which ends at vertex 24, 12% inside the top vertex on the blue line).

Of course, the optical illusion is more pronounced with the smaller sided n-gons (like the bottom row) in which each vertex is 6/200 = 3%, or 4/200 = 2%, or 3/200 = 1.5%, inside the last time that parent vertex was used.

One other thing to note here is that only $1/J$ of the parent polygon vertices are used in constructing each image. Each endpoint (where two segments meet) is on one of α straight lines to the center of the circle. This is readily seen by noting that all segment endpoints are on one of α vertex rays (each used ray is separated from other used vertices by J arcs). In each instance and the other n-α vertices remain unused. You can see this by noting that there are J-1 red dots (or unused vertex rays) between each used red dot in each image.

Finally, it is worth noting that none of these almost polygons are regular. Indeed, lines on a given side are not parallel to one another. This is easiest to see using smaller values of r like the $r = 10$, $n = 9$, $J = 3$ almost triangle in *explainer* 10.1b.