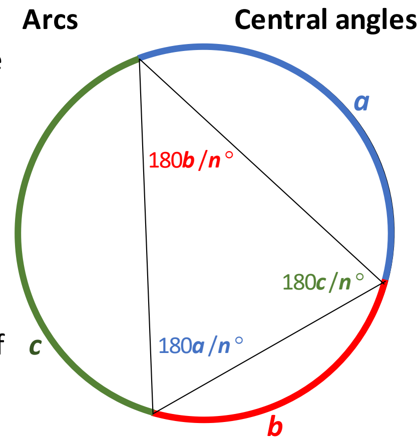
Glossary of Commonly Used Terms: Cardioids
(Reference to where the term was introduced)

Arcs of a Circle and *Central Angles*: Connecting any three vertices of a regular $\boldsymbol{n}$-gon produces a triangular image. One can easily describe the size of the angles of this triangle by counting the number of vertices between points, $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$. By construction, $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{n}$ in the image to the right. One can think of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ as *arcs of a circle* that cumulatively create the circle. Put another way, the *central angles* created by connecting endpoints of each arc to the center of the circle sum to 360°, each arc has central angle of $360\boldsymbol{a}/\boldsymbol{n}°$, $360\boldsymbol{b}/\boldsymbol{n}°$, and $360\boldsymbol{c}/\boldsymbol{n}°$. The triangle angles are $180\boldsymbol{a}/\boldsymbol{n}°$, $180\boldsymbol{b}/\boldsymbol{n}°$, and $180\boldsymbol{c}/\boldsymbol{n}°$. Each of these angles is an example of an *inscribed angle*.



Adjacent interior angles: These are the angles created when two adjacent vertices are the starting points of line segments. If these lines intersect on the interior of the parent $\boldsymbol{n}$-gon the angle created by their intersection is $180(k+1)/n°$.

Cardioid line creation rule: Create the image one line at a time by connecting each starting vertex, $\boldsymbol{j}$, $1 \le \boldsymbol{j} < \boldsymbol{n}$ to its multiple, $\boldsymbol{k}\cdot\boldsymbol{j}$ where the ending vertex is the remainder of $\boldsymbol{k}\cdot\boldsymbol{j}$ once divided by $\boldsymbol{n}$. (11.1)

Circle fan: A circle fan involves having a number of equally spaced vertices connecting to a single vertex. That vertex is called the circle fan base. (11.7)

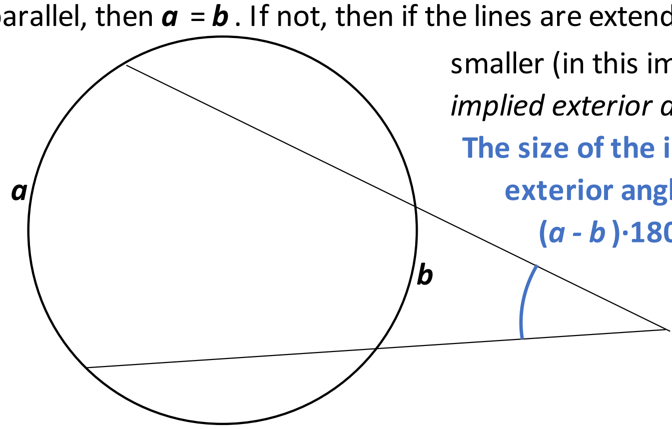
Central angle: See Arc above.

Cusp: A cusp is a gathering point of lines used to create the cardioid (11.1)

Degree of rotational symmetry: The degree of rotational symmetry, $\boldsymbol{G}$, is the number of different vertices it can be rotated to for which it matches the original. $\boldsymbol{G} = \text{GCD}(\boldsymbol{n}, \boldsymbol{k}-1)$. (2.5c and 11.2b)

GCD: The greatest common divisor of two numbers is the largest factor common to both.

Implied Exterior Angle: Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be the number of vertices between intersection points**.** If the lines are parallel, then $\boldsymbol{a} = \boldsymbol{b}$. If not, then if the lines are extended, there will be an intersection point on the side that is smaller (in this image, $\boldsymbol{b} < \mathbf{a}$). Call the angle created by these lines the *implied exterior angle.*



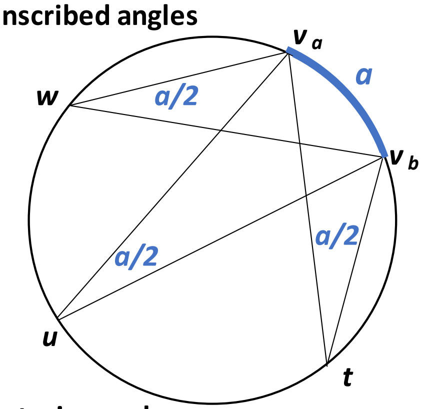
The size of the implied exterior angle is $(\boldsymbol{a} - \boldsymbol{b})\cdot 180/\boldsymbol{n}°$

Identity vertices: If the starting and ending vertex of a line coincide, the vertex is called an *identity vertex*. An identity vertex is a vertex loop of length one.

Intersecting line segments create angles in one of two ways: *Inscribed Angles* and *Interior Angles.*

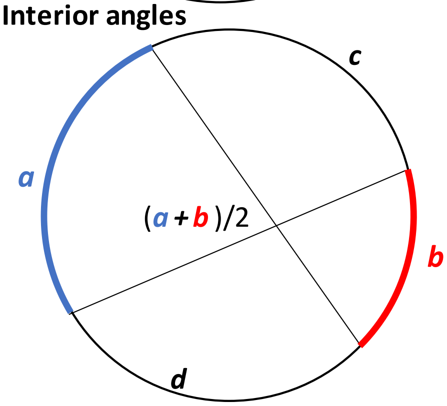
Inscribed Angles: If an angle is formed from 3 points on a circle, the angle formed is called an inscribed angle. If the three points are vertices of a regular n-gon then the angle created isdetermined by the number of vertices between points. In the graph, let $\boldsymbol{a} = \boldsymbol{v}_a - \boldsymbol{v}_b$ where $\boldsymbol{v}_a$ and $\boldsymbol{v}_b$ are vertices of the regular ***n-*** gon. The angle created using these vertices as ends of the legs will be the same regardless of where the third vertex is on the circle. The image shows three such examples at points ***t*** , ***u*** and ***w*** on the perimeter of the circle. The angle in each case is half the size of the central angle according to the *Inscribed Angle Theorem.* This angle is $180\boldsymbol{a}/\boldsymbol{n}°$.

Inscribed angles



Interior Angles: Interior angles are created when two segments intersect on the interior of the circle. Four angles are created but only two of them are distinct. If ***a*** and ***b*** represent the two opposing arcs then the angle created is $(\boldsymbol{a}+\boldsymbol{b})/2$ as shown to the right. However, if ***a*** and ***b*** represent the number of vertices between regular ***n*** -gon vertices, the angular measure is $(\boldsymbol{a}+\boldsymbol{b})\cdot 180/\boldsymbol{n}°$

Interior angles



Note that had we examined the two other arcs created by these two lines, ***c*** and ***d*** , then the measure of the angle created from ***c*** and ***d*** , $(\boldsymbol{c}+\boldsymbol{d})/2$, is supplementary to $(\boldsymbol{a}+\boldsymbol{b})/2$. Supplementary angles sum to 180°, and since $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}+\boldsymbol{d} = 360°$, $(\boldsymbol{a}+\boldsymbol{b})/2 + (\boldsymbol{c}+\boldsymbol{d})/2 = 180°$.

If line segments do not intersect on the edge or interior of the polygon, their extensions will intersect on the exterior of the polygon unless the lines are parallel. When this exterior intersection occurs, we can describe how far apart the lines are on an angular basis using the *implied exterior angle* .

k: ***k*** is the multiplying factor connecting starting vertex to ending vertex for each vertex in the polygon.

Looped vertices: A *vertex loop* is a set of vertices in which the end of the last vertex in the loop is the start of the first vertex. Vertex loops are closed sets of vertices. The length of the loop is the number of distinct vertices in the loop. (11.6)

MOD: The remainder when one number is divided by another is labeled as ***r*** = MOD(***a***, ***b***). This means that ***a*** can be written as $\boldsymbol{a} = \boldsymbol{b}\cdot\boldsymbol{c} + \boldsymbol{r}$ where $\boldsymbol{r} < \boldsymbol{b}$ and ***c*** is a whole number.

Paired vertices: When two vertices form a loop, the vertices are called *paired*. Paired vertices are a 2 vertex loop.

Ribbon: Ribbons are narrow sets of parallel lines created when lines are the same small number of vertices between each set of end points.

Rotational symmetry: See Degree of rotational symmetry.

Spotlights: Sharp angles originating at a polygonal vertex surrounded by less-sharply created intersections. For example, ***n*** = 44, ***k*** = 6 gives 1 arc spotlights at 14 and 30.

String: A string is a set of 1 or more vertices that leads into a loop but is not part of that loop. In ***n*** = 360, ***k*** = 2, vertices 1, 3, and 5 start 3-vertex strings. The string starting at 1 connects to 2 and 4 before connecting with 8 which is part of a 12-vertex loop. By contrast, 3 connects to 6 and 12 before connecting with 24 which is part of a 4-vertex loop and 5 connects to 10 and 20 before connecting with 40 which is part of a 6-vertex loop.