

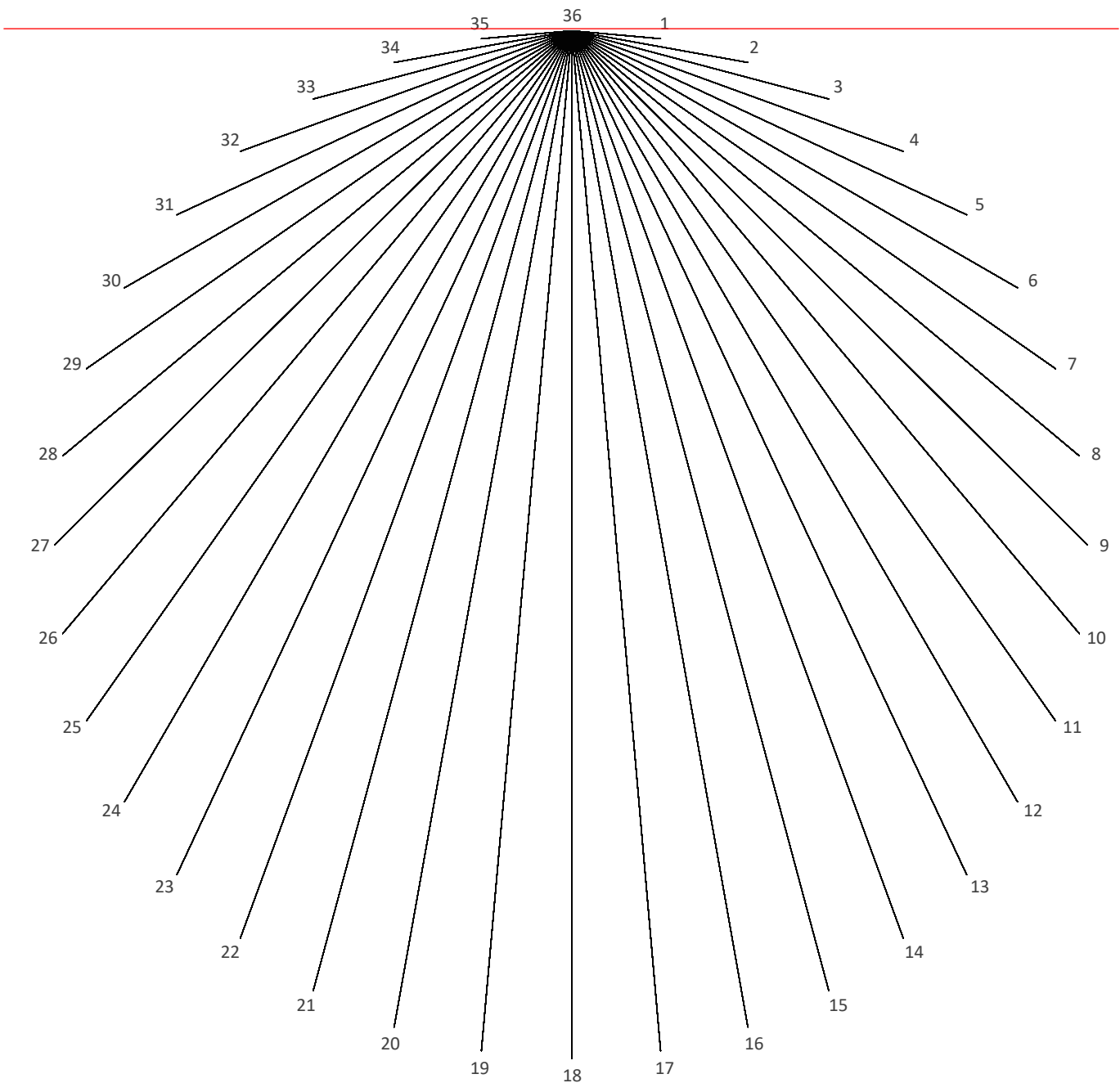
n Vertices Implies n Possible Directions (angles relative to the horizontal line)

Consider the cardioid-produced circle fan given $n = 26$ shown below. This shows 35 lines, one starting at each vertex 1 – 35 and ending at 36. Each line is 5° steeper than the previous line because each angle created is $180/36 = 5^\circ$. There are 34 such angles since angles are drawn as $j-36-j+1$ for $j = 1, \dots, 34$ (with the dash meaning line between). This means that angle 1-36-35 is 170° . (The remaining 10° are 5° on either side of the **horizontal line touching vertex 36 shown in red.**)

These 35 lines can be thought of as describing 35 of the 36 possible directions available for ANY image given $n = 36$ (the 36th direction is horizontal). We could use the numbers j from 0 to $n-1$ to describe these directions (where **0 means horizontal**). Each j can also be thought of in angular terms as $180j/n^\circ$ clockwise relative to horizontal.

So, to use the image below, if we draw lines from vertex 35 to 2, or 34 to 3, or 33 to 4, or, ..., or 19 to 18, all would be parallel to the line we call 1 (from 36 to 1). This idea is the basis for [PART II of PwP, Parallel Lines](#).

We can use this same idea to talk about line segments in general given cardioid models. An example is given on the next page. Doing so will allow us to have another way to describe patterns in the cardioid images we find using n and k .

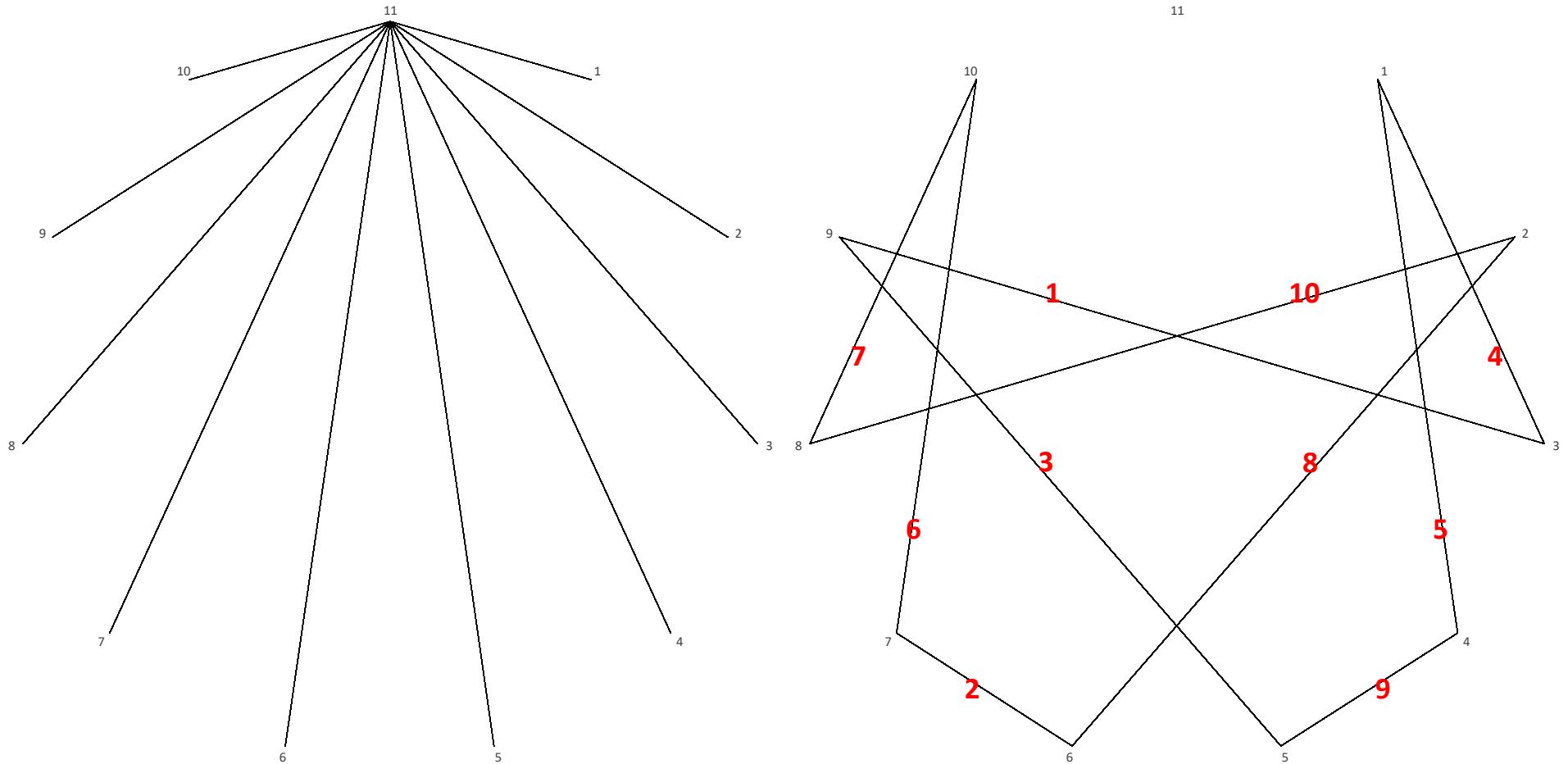


Both images below are based on $n = 11$. The left shows the circle fan produced when $k = 11$ and the second image occurs if $k = 3$ or $k = 4$. Both images show 10 lines with **THE SAME** 10 different directions (or angles relative to horizontal). The image on the right uses the idea that you can describe the direction of a line by finding the line parallel to that line going through the top vertex (here 11). This direction is shown using a red number ($1 - 10$) touching each line segment.

To explain further, we denote each vertex lines by $a-b$ where the first number, a , is closer to the top vertex (11).

If $a < n/2$ we add a to b to find $\#$ as is true for these five segments: 1-3 is **4**; 1-4 is **5**; 2-6 is **8**; 2-8 is **10**; and 4-5 is **9**.

If $a > n/2$ we subtract $n-a$ (here $11-a$) from b to find $\#$ as is true for these five segments: 10-8 is **7**; 10-7 is **6**; 9-5 is **3**; 9-3 is **1**; and 7-6 is **2**.



One can immediately see that these red directions come in pairs. (These pairs are presented at different heights from one another to highlight this pairing.) Due to the vertical symmetry inherent in all cardioid images, each pair supplements one another and sums to n (here 11).