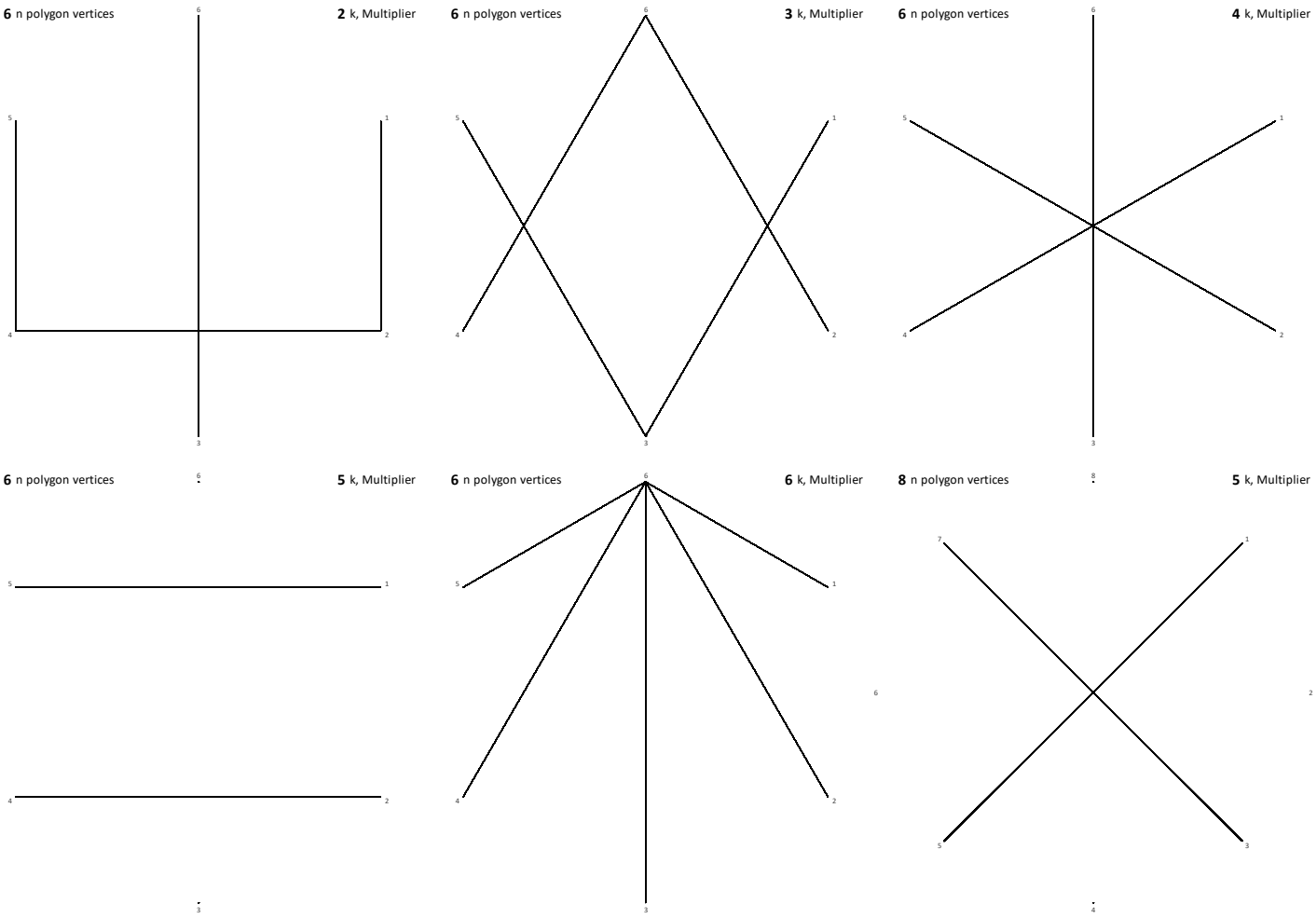
# 11. Number of Lines in an Image

Cardioid lines are created according to the rule: Connect a line from each vertex of a regular ***n***-gon ***v***, $0 \le v \le n-1$ to vertex $k \cdot v$ where ***k*** is a whole number exceeding 1. If $k \cdot v > n-1$, the ending vertex ***w*** is the remainder upon division by ***n***, $0 \le w < n$, and $k \cdot v = m \cdot n + w$ where and ***m*** is a whole number, or $w = \text{MOD}(k \cdot v, n)$ where MOD is the remainder function.

Given this, one might expect the image would have ***n*** lines, one for each vertex. Explainer 11.1a notes that there will never be a line starting at 0 because the ending vertex is also 0. By counting the lines from where they start, we see that there are AT MOST ***n***-1 lines in an image. The six images below show the range of possibilities. The first five are based on ***n*** = 6 for ***k*** from $2 \le k \le 6$, the last shows an ***n*** = 8 example. From left to right, there are 3, 4, and 3 lines (top) and 2, 5, and 2 lines (bottom). The bottom middle circle fan has ***n***-1 lines and the other two bottom images have the fewest lines.




**Minimal density.** The ***n*** = 8, ***k*** = 5 image has two lines, one fourth of the vertices. This is the smallest percentage of lines that are possible. It occurs when half of the vertices are identity vertices (0, 2, 4, and 6) and the remaining half are paired vertices (1 with 3 and 3 with 7). More generally, this occurs if $n = 2(k-1)$ and ***k*** is odd. In this instance, opposing odd vertices connect with one another (are paired) and even vertices have no line.

**Maximal density.** Counting lines may lead to confusion as ***n*** becomes large but there are rules of thumb to use. The circle fan and the image to the right both have ***n***-1 lines. Note that each of the ***n***-1 = 10 vertices other than the top have 2 lines at each vertex. Since a segment has a starting vertex and an ending vertex, this means there are 10 lines in the image. Whenever you find an image where all vertices appear to have two lines, the image has ***n***-1 lines.