## Further Automating the Functional Relation $n(k)=\boldsymbol{a} \boldsymbol{k}+\boldsymbol{b}$

Explainer 11.5a showed how to connect $\boldsymbol{n}$ to $\boldsymbol{k}$ via an equation in cell B1. This explainer provides a quick way to automate that equation so that you can easily adjust it and see the results.

We consider here a general relation $\boldsymbol{n}(\boldsymbol{k})=\boldsymbol{a} \boldsymbol{k}+\boldsymbol{b}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are whole numbers (which can be negative or positive). (A negative value of $\boldsymbol{n}$ simply means counting around the circle in a counterclockwise direction.)

Automating $\boldsymbol{n}(\mathbf{k})$. Instead of clicking on cell B1 and changing the equation each time you want to consider a new functional relation between $\boldsymbol{n}$ and $\boldsymbol{k}$, we can use the unprotected green cells starting in P1 to help simplify equation adjustments. The idea is to put $\boldsymbol{a}$ and $\boldsymbol{b}$ in cells (P12 and P13) then link the equation for $\boldsymbol{n}$ (in B1) to BOTH $\boldsymbol{k}$ in D1 and $\boldsymbol{a}$ and $\boldsymbol{b}$ in P12 and P13. (The equation to type in B1 is shown in cell P14 below.)

| Column |  | $\cdots$ | $\mathbf{P}$ | $\mathbf{Q}$ |
| :---: | :---: | :---: | :---: | :---: |
| Row | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| 11 | $\cdots$ | $\cdots$ | Equation for cell B1: $\boldsymbol{n}=\boldsymbol{a} \boldsymbol{k}+\boldsymbol{b}$ |  |
| 12 | $\cdots$ | $\cdots$ | $\mathbf{3}$ | $\boldsymbol{a}$ (in cell P12) |
| 13 | $\cdots$ | $\cdots$ | $\mathbf{3}$ | $\boldsymbol{b}$ (in cell P13) |
| 14 | $\cdots$ | $\cdots$ | In B1 type: $\boldsymbol{= P 1 2 *}$ D1+P13 enter. |  |
| 15 | $\cdots$ | $\cdots$ | Then use $\boldsymbol{k}$ arrows in C1:C2. |  |

Once you type in this equation in B1, all you do is change $\boldsymbol{a}$ or $\boldsymbol{b}$ to order to test a new relation. Consider the three functions examined in 11.5a, $\boldsymbol{n}=\boldsymbol{k}, \boldsymbol{n}=\mathbf{2 k}$, and $\boldsymbol{n}=\mathbf{2 k}+1$. The first require 1 in P12 and 0 P 13 . Change P12 to 2 and you see the second. Change P13 to 1 and see the third ... all without touching B1.

The images below show the $\boldsymbol{a}$ and $\boldsymbol{b}$ suggested above, $\boldsymbol{n}=3 \boldsymbol{k}+3$ for $\boldsymbol{k}=6$ and $\boldsymbol{k}=7$. As $\boldsymbol{k}$ changes, one still sees only equilateral triangles, but other patterns emerge as well. In particular, see if you can identify different patterns based on the remainder of $\boldsymbol{k}$ once $\boldsymbol{k}$ is divided by 6 . The two images below show remainder 0 and 1. For example, when $\boldsymbol{k}$ is one more than a multiple of 6 (like the right image below) one can see regular hexagons). Every other version will show 6 pie pieces (like below). but $\boldsymbol{k}=13$ has no such inner pie pieces (however $\boldsymbol{k}=19$ once again looks like an extended version of $\boldsymbol{k}=7$ below).


