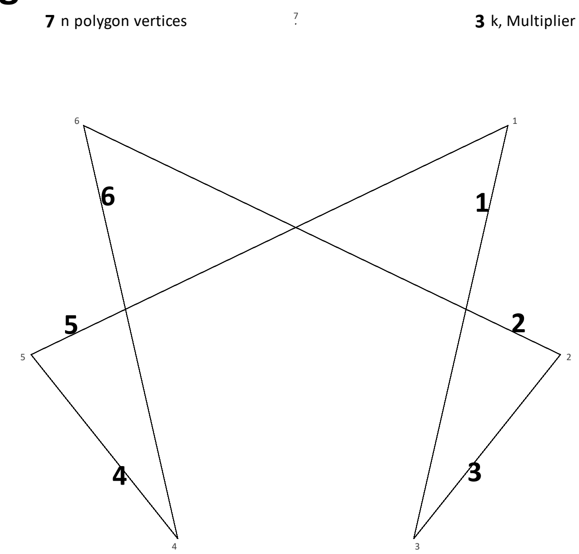
Implied Exterior Angles

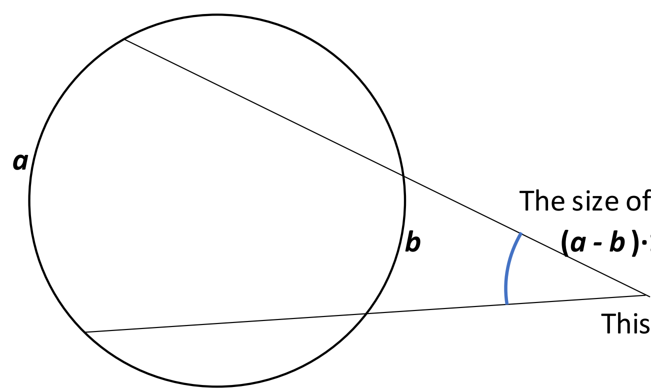


The image to the right has six non-parallel line segments. These lines are labelled **1**-**6** using the cardioid line creation rule as a guide.

Any two lines that are not parallel must intersect so that the total number of intersections is 15. (As an aside: 15 = 6·5/2 using the sum of numbers counting rule discussed in File 6 which applies because **1** intersects with **2**-**6**, **2** intersects with **3**-**6**, **3** intersects with **4**-**6**, **4** intersects with **5**-**6**, and **5** intersects with **6**, or 5+4+3+2+1 = 15.)

These intersections can occur in one of three locations relative to the vertices forming the endpoints of the segments, two of which are visible in the images created here: interior intersections, (3); and intersections at vertices, (6). The remaining 6 intersections are the third type of intersection. These occur only by considering extensions of these segments. These extensions would eventually intersect because two lines always intersect unless they are parallel. Such intersections are exterior to the image (these are intersections: **1**-**4**, **1**-**6**, **2**-**4**, **3**-**4**, **3**-**5**, and **3**-**6**). We call the angles created by these intersections, implied exterior angles. We use "implied" because the angles are not in the image.

How large are implied exterior angles? Let ***a*** and ***b*** be the number of vertices between intersection points. If the lines are parallel, then ***a*** = ***b***. If not, then if the lines are extended, there will be an intersection point on the side that is smaller (in this image, ***b*** < ***a***). Call the angle created by these lines the implied exterior angle.

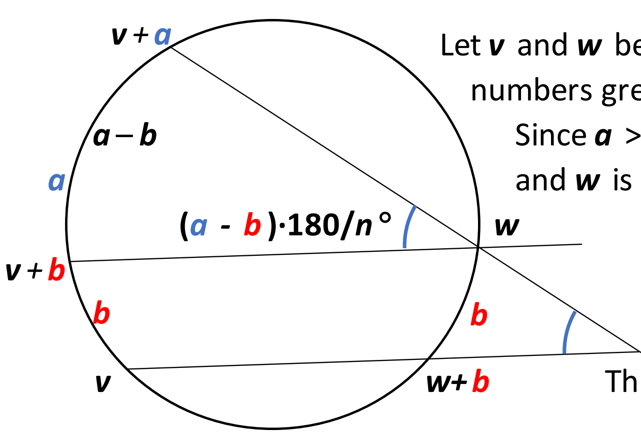


The size of the implied exterior angle is

(*a* - *b*)·180/*n* °

This angle is called the *implied exterior angle*

Justification of this formula using the parallel lines created with regular polygonal vertices.



Let ***v*** and ***w*** be vertices of an ***n***-gon and ***a*** and ***b*** are numbers greater than zero where ***a*** + ***b*** < ***n*** - 1 and ***a*** > ***b***. Since ***a*** > ***b***, we can add ***b*** to ***v*** and the line between ***v*** + ***b*** and ***w*** is parallel to the line between ***v*** and ***w*** + ***b***. This newly constructed line has the same angle as the origional angle but this angle is an inscribed angle. This angle has size (***a*** - ***b***)·180/n°.

This angle is called the *implied exterior angle*

(We restrict ***a*** + ***b*** < ***n*** - 1 because we want at least one vertex between ***w*** + ***b*** and ***v*** as well as one between ***v*** + ***a*** and ***w***.)

In the top image: the **1**-**6**, **2**-**4** and **3**-**5** angles are 180/7°; the **1**-**4** and **3**-**6** angles are 360/7°; and the **3**-**4** angle is 540/7°.