11. Vertex Loops

Images in the Cardioid file are created one line at a time starting at vertex 1 and ending at vertex n-1. Each starting vertex v is connected to an ending vertex w = MOD($k \cdot v$, n). Individual lines need not appear connected to one another, but they may appear that way. Consider the three images n = 11, 12 and 13 with k = 9.

The middle n = 12, 8 line image is composed of 4 V-shaped pieces. The middle of each V is at a vertex which is a multiple of 3. These midpoint vertices are endpoints of each line segment, but not the starting point of any vertex. Consider the 1-9-5 V:

Starting point 1 ends at 9 = 1·9.
Starting point 5 ends at 9 = MOD(5·9, 12) since 45 = 3·12 + 9.
Starting point 9 ends at 9 = MOD(9·9, 12) since 81 = 6·12 + 9.

In this instance, one cannot create the image moving from vertex to vertex in a manner similar to the connected segments in **PART I**.

Vertex loop. Define a set of *looped vertices* (or a *vertex loop*) as one in which the end of the last vertex in the loop is the start of the first vertex. *Vertex loops create closed sets of vertices*.

The upper n = 11, 10 line image can be conceptualized as being created in two loops of 5 vertex jumps following the cardioid line creation rule, simply applied in a different fashion. Denote these jumps with → so that the two loops can be described as:

1 → 9 → 4 → 3 → 5 → 1 and 2 → 7 → 8 → 6 → 10 → 2 .

One could have started at any of vertices in each loop and the connected set of lines would remain the same because the end of the last jump is the start of the first.

We call each such grouping a 5-vertex loop.

You may check that if you reduce k by 1 to k = 8 and n = 11, the image is a single 10-vertex loop. This image is not shown in this explainer but starting at vertex 1, the loop looks like :

1 → 8 → 9 → 6 → 4 → 10 → 3 → 2 → 5 → 7 → 1 .

The bottom n = 13, 12 line image has 4 triangles. Put another way, there are four 3-vertex loops in this instance:

1→ 9→ 3→ 1; 2→ 5→ 6→ 2; 4→ 10→ 12→ 4; 7→ 11 8→ 7 .

What happens if there are two lines at each vertex? If each vertex except the top vertex has 2 lines, then one line starts, and the other ends at each vertex. The top and bottom images have two lines at each vertex. In this instance, the image is created from one or more sets of Vertex loop. Note that the same image is created using two k values less than n, one for each of the ending points of the lines at vertex 1. The same cannot be said if the number of lines at vertex 1 is different from 2.

11 n polygon vertices   **9** k, Multiplier

12 n polygon vertices   **9** k, Multiplier

13 n polygon vertices   **9** k, Multiplier