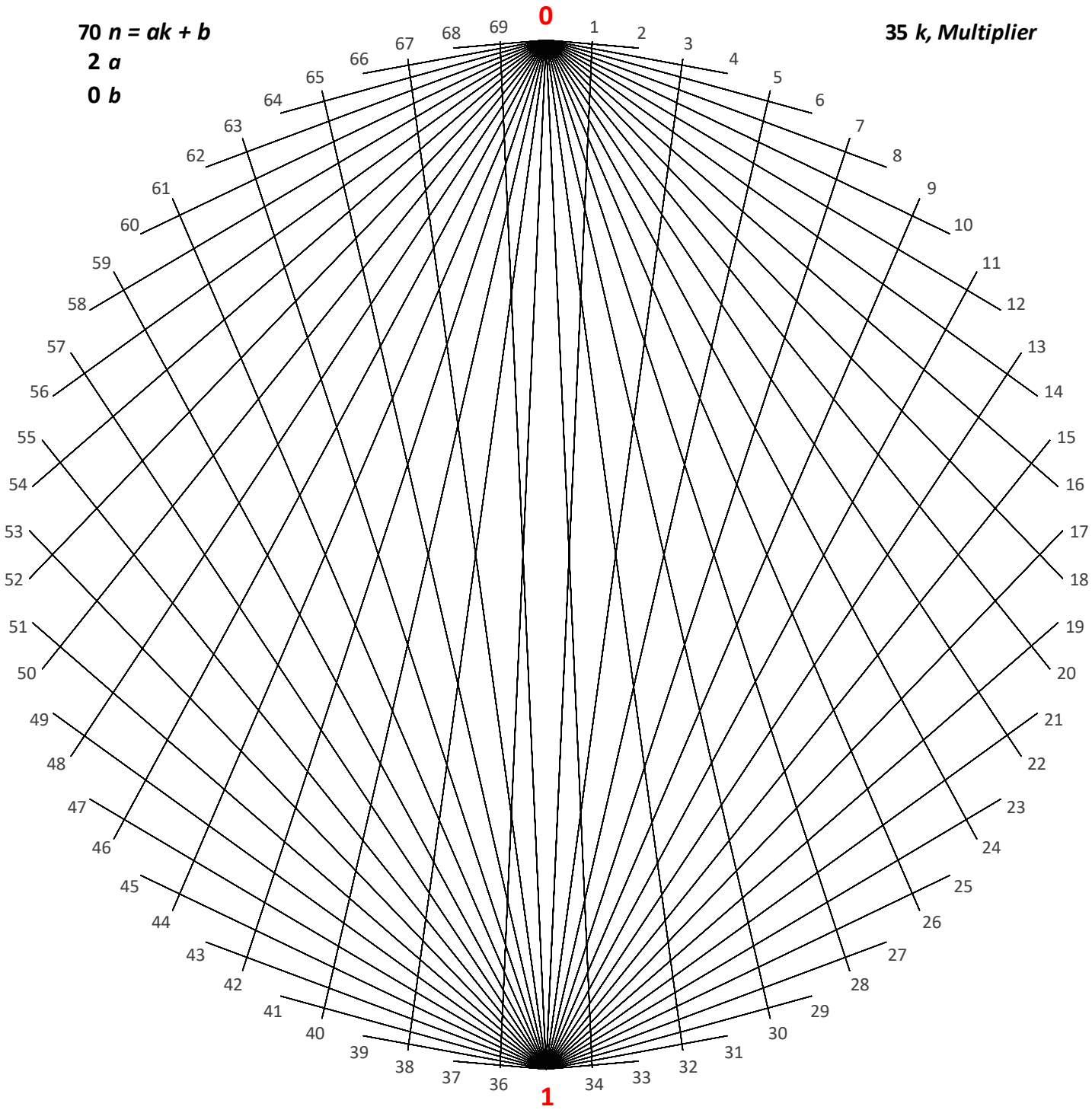
11. Each Circle Fan shows a Different Remainder

Any image where ***n*** is a multiple of ***k***, so we can write ***n*** = ***a***·***k*** where ***a*** is a whole number, will have ***a*** equally spaced circle fans around the image. The endpoints of the top circle fan are divisible by ***a*** (have remainder **0** upon division by ***a***). Moving clockwise around the image, each subsequent circle fan has remainder 1 higher than the last with the last having remainder ***a***-1. Each circle fan base is the next higher multiple of ***k***. These multiples are also the remainders and are noted as **bold red**. Two examples are provided here, both are based on ***n*** = 70 but differ in their values of ***a*** and ***k***.

The simplest example, ***n*** = 2***k***, has ***a*** = **2** circle fans one with base at the top (remainder **0**) and the other with base at the bottom of the polygon (remainder **1**). Note that an odd vertex can be written as ***V*** = 2***b***+1 and even as ***V*** = 2***b***.

Odd start: End vertex is 35·(2***b***+1) = 70***b*** + 35. 70***b*** is a multiple of ***n*** = 70; the ending vertex is 35 = ***n***/2, circle fan base **1**.
Even start: End vertex is 35·(2***b***) = 70***b***, a multiple of ***n*** = 70 so the ending vertex is at 70 = ***n***, circle fan base **0**.

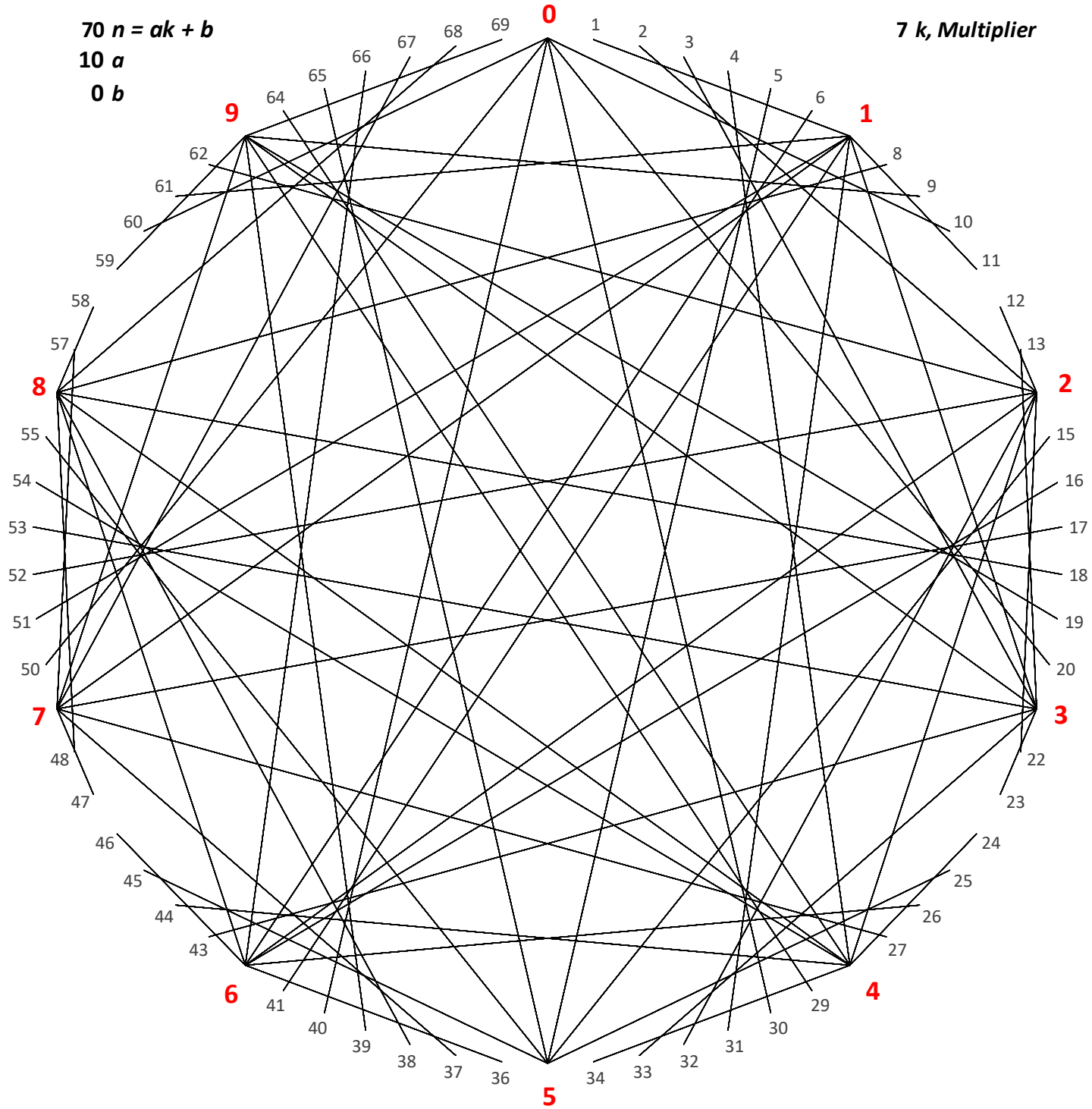


The even/odd pattern noted here is simply a result of any number having remainder 0 or 1 upon division by ***a*** = 2.

The second example, ***a*** = **10**, has equally strong intuitive underpinnings because the remainder of any number upon division by 10 is simply the last digit of the number. In the example below, ***k*** = 7 so ***n*** = 70 (just like the first image).

This image has ***k***-1 = 6 cusps and ***a*** = 10 circle fans. A quick glance at the **0** and **5** circle fan endpoints confirms that all end in 0 and 5 respectively. These endpoints are, in fact, the starting points of the line segments which end at **0** or **5**.

A visual inspection of other remainders shows that sometimes the pattern appears to be broken. For example, consider the lines sharing **1** as a base. One of those lines connects vertex 7 to 49 and 49 does not have last digit 1. This particular line STARTED at 7 (the base for the remainder **1** circle fan) and ended at 49 which is the base of the remainder **7** circle fan. Further inspection will yield additional example where there is an additional line at a circle fan base but that is because that base is also the starting vertex of a line which has as its endpoint ***k*** times that vertex.



a circle fans will exist whenever ***n*** = ***a***·***k*** but it often will be more challenging to see the resulting remainder patterns because our senses are not as attuned to readily recognize remainders of bases other than 10 (last digit) or 2 (even/odd).