Two Special Cases of Vertex Loops: *Identity Vertices* and *Paired Vertices*

A set of vertices is said to be *looped* if the end of the last vertex in the loop is the start of the first vertex. This explainer examines the two smallest loops, 1 and 2, because both appear different from loops with more than 2 vertices. From the start of our discussion of cardioids, we have noted that the top vertex always starts and ends at the top.

If the top vertex has a line (or more than one), it did not start there! Although it seems like the *cardioid creation rule* (connect each starting vertex v to an ending vertex w by constant multiple of that vertex $k \geq 2$, with $k \cdot v = m \cdot n + w$ where m is a whole number and $0 \leq w < n$) should create n lines, one for each vertex of the n-gon, the most that are ever visible is n-1. The top vertex can be called 0 or n. In either event, if this is the starting vertex, it is also the ending vertex because $0 = \text{MOD}(k \cdot v, n)$ regardless if $v = 0$ or $v = n$. This is why we focused on vertices 1 to n-1 in *explainer* 11.1a.

***Identity vertices* start and end at the same vertex.** Using the terminology of explainer 11.6a, an identity vertex is a loop of length 1. The top vertex is only one example of an identity vertex. If $v = \text{MOD}(k \cdot v, n)$, then the starting vertex and the ending vertex are the same and we say that v is an *identity vertex* (much like multiplying by 1 is an identity or adding 0 is an identity). No line is drawn starting at an identity vertex (but many lines may end there as the circle fan *explainer* 11.5 makes clear).

The top vertex is always an identity vertex, but other vertices can be as well. Indeed, if $n = k$-1, ALL vertices are identity vertices and there is no image (or more accurately, the image has no lines but is simply the n vertices as points). In all other instances, there will be some lines in the image. If v is an identity vertex, then $k \cdot v = m \cdot n + v$. Regrouping we see that $(k-1) \cdot v = m \cdot n$, or $(k-1) \cdot v$ is a multiple of n. If the image has rotational symmetry of degree G ($G = \text{GCD}(k-1, n)$), then every n/G vertices will be an identity vertex. The left image has $G = 3$ and every $15/3 = 5^{th}$ vertex is an identity vertex, but the right image has $G = 4$ and every $16/4 = 4^{th}$ vertex is an identity vertex.

Paired vertices. When two vertices form a loop then the vertices are called paired vertices. All six lines in the left panel, and the lines connecting vertices 2 to 10 and 6 to 14 in the right panel are all examples of paired vertices. One can test for identity and paired vertices using the yellow cell N5 in the Cardioid file.

Image having 6 paired vertices and 3 identity vertices

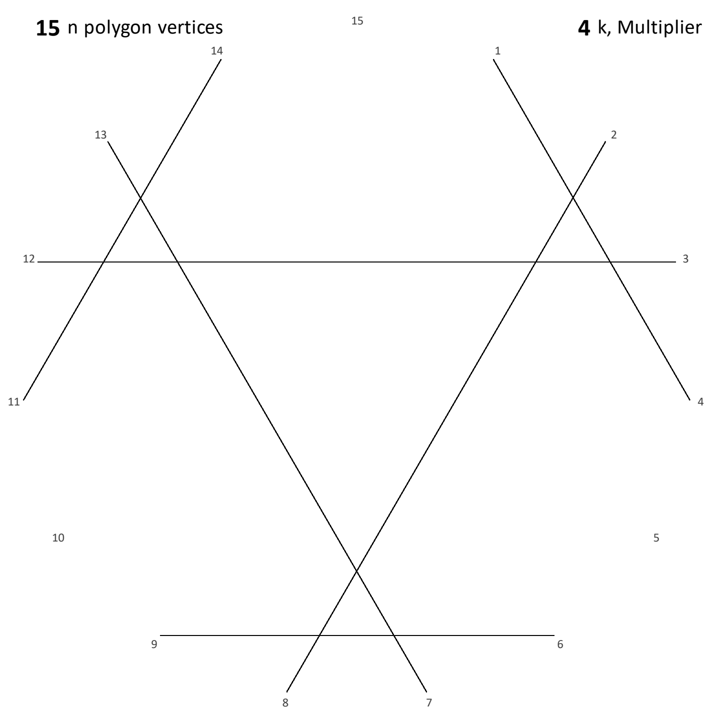

Image with two 4-vertex loops, 2 paired vertices and 4 identity vertices