11. Three Special Cases (when n and k are close to one another)

Why focus on k ≤ n? The Cardioid file allows $3 \le n \le 360$ and $2 \le k \le 360$ but, as a practical matter, one does not obtain different results when $k > n$ because when this is true, we can use another smaller value in place of k to produce the same image. In particular, let r be the *remainder* upon division by n, so that $k = m \cdot n + r$ where m is the largest whole number multiple of n and $r < n$. Images created using k and r will be the same because all that matters (from the perspective of line placement) is the remainder upon division by n.

Despite this, it is worth considering what happens when k and n are very close to one another including when k is a bit larger than n.

$k = n - 1$. All lines are horizontal and connect vertex v with $n - v$. Take vertex v and multiply by $k = n - 1$.

Starting vertex is v. Ending $k \cdot v = (n - 1) \cdot v = n \cdot v - v = n \cdot v - n + n - v = n \cdot (v - 1) + [n - v]$.

Starting vertex is $n - v$. Ending $k \cdot (n - v) = (n - 1) \cdot (n - v) = n^2 - n - n \cdot v + v = n \cdot (n - v - 1) + [v]$.

The $n = 12$, $k = 11$ image is shown on the left. In this instance, vertices v and $n - v$ are *paired vertices*, the top is always an *identity vertex* and when n is even, so is the bottom (see *explainer* 11.6b).

$k = n$. All lines end at the top (vertex 0) since $k \cdot v = n \cdot v = n \cdot v + [0]$ for all v because all vertices are multiples of n. This is a *circle fan* (discussed at greater length in *explainer* 11.7a). The $n = k = 12$ circle fan is shown on the right.

$k = n + 1$. In this instance all ending vertices are the same as starting vertices.

Starting vertex v: Ending $k \cdot v = (n + 1) \cdot v = n \cdot v + v = n \cdot v + [v]$

Put another way, the image can be thought of as n points but no lines. In this instance, all vertices are identity vertices (see *explainer* 11.6b). This is why the smallest k considered is $k = 2$. This (empty) image is not shown but you can readily check out what happens yourself using the Cardioid file.