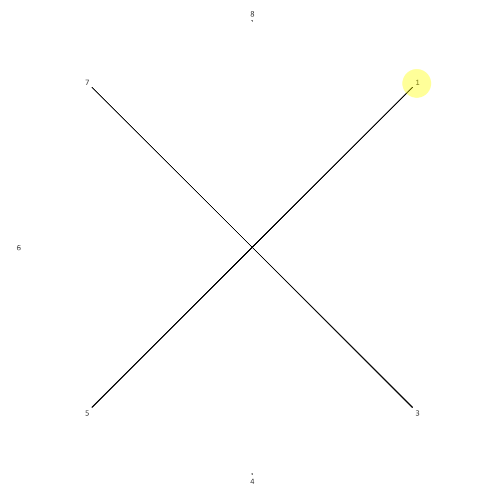
11. To Analyze an Image, Start at Vertex 1

If you want to understand why images look the way they do in the Cardioid file, it is worthwhile to start at vertex 1. There are a couple of reasons for this assertion. To understand why, start with how lines are created in the first place in these images.

Cardioid line creation rule. All images are created as a result of considering each vertex, $\boldsymbol{v}$, of the $\boldsymbol{n}$-gon as the starting vertex of a segment where the ending vertex $0 \le \boldsymbol{w} < \boldsymbol{n}$ where $\boldsymbol{w}$ is the remainder once $\boldsymbol{k} \cdot \boldsymbol{v}$ is divided by $\boldsymbol{n}$. This is noted mathematically as $\boldsymbol{k} \cdot \boldsymbol{v} = \boldsymbol{m} \cdot \boldsymbol{n} + \boldsymbol{w}$ where $\boldsymbol{m}$ is a whole number and $\boldsymbol{w} < \boldsymbol{n}$, or using the MOD function as $\boldsymbol{w} = \mathrm{MOD}(\boldsymbol{k} \cdot \boldsymbol{v}, \boldsymbol{n})$ where MOD is the remainder function. As noted in *explainer* 11.1c, we can restrict ourselves to $2 \le \boldsymbol{k} \le \boldsymbol{n}$.

Reason 1. The ending vertex is easy to spot if $\boldsymbol{v} = 1$. This follows from the fact that $\boldsymbol{k} \cdot 1 = \boldsymbol{k}$ and we need not worry about remainders since the largest $\boldsymbol{k}$ that need be considered to see all distinct images is $\boldsymbol{n}$. When $\boldsymbol{k} = \boldsymbol{n}$, every vertex ends at the top vertex (as discussed in *explainer* 11.1c). For all other values of $\boldsymbol{k}$, the ending vertex of the line starting at vertex 1 is smaller than $\boldsymbol{n}$ and hence $\boldsymbol{k}$ is the same as the remainder upon division by $\boldsymbol{n}$. For all other vertices, this need not be the case because $\boldsymbol{k} \cdot \boldsymbol{v} > \boldsymbol{n}$ is possible if $\boldsymbol{v} > 1$.



Reason 2. There is always at least one line with endpoint at vertex 1. The endpoint will NOT be at 1 if the starting point is 1 since $2 \le \boldsymbol{k} \le \boldsymbol{n}$. Indeed, the only way for vertex 1 to not have a line is to have $\boldsymbol{k} = \boldsymbol{n}+1$ or a multiple of $\boldsymbol{n}+1$. In this instance, there is no image as all vertices are identity vertices (*explainer* 11.1c). The $\boldsymbol{n} = 8$ image to the right shows that this is not the case at vertex 2.

Reason 3. By its very nature, a line segment has two endpoints. Since all line segments end on vertices of the $\boldsymbol{n}$-gon, any line segment touching a vertex must be there because it is either the ending point of a segment or the starting point of a segment.

When multiple segments share an endpoint, AT MOST one of those segments acts as its starting point. The middle of the $\boldsymbol{n} = 10$ examples below shows that this need not be the case. Vertices 10 and 5 both have 4 segments but all 4 are endpoints and none is a starting point since 5 and 10 are identity vertices. The right image has 3 segments at vertices 2, 4, 6 and 8 but in each case, two of those segments are ending points and one is a beginning point.

Reason 4. If two lines share an endpoint at 1 then if we call the other endpoint $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$, both produce the same image using those values as the multiplication factor $\boldsymbol{k}$. The $\boldsymbol{n} = 10$ left image can therefore be created using either $\boldsymbol{k} = 3$ or 7. The same cannot be said for other vertices.

Reason 5. If there is only one line at vertex 1, the other end is $\boldsymbol{k}$. So, $\boldsymbol{k} = 5$ above, and below, $\boldsymbol{k} = 5$ in the middle image and $\boldsymbol{k} = 8$ to the right.