# 11. Ribbons and Strips

Ribbons are sets of parallel lines that wrap back and forth across the image. Parallel lines are created whenever two lines are the same number of vertices "apart" from one another. One can image the area between these lines as a *strip*.

**Width.** The width of a strip can be thought of in terms of how many vertices apart are the parallel sides. The narrowest strips are 1-vertex wide (this "distance" depends on where endpoints lie relative to one another (comparing the yellow 1-wide strips shows that those with closer endpoints (2 for 25-27 versus 17 for 37-15) create narrower strips).

**Folds.** Not all images with parallel lines (strips) have ribbons. *To be a ribbon, the set of strips must have at least one **fold*** (or set of vertices common to both strips). Imagine folding the ribbon at these vertices. The 1-2 vertex in the bottom image is one example of such a fold (that image has a total of 13 folds).

Compare the image top with the bottom image. Both are based on $n = 3k + 3$. Both have several 1-wide strips but the top ($n = 39$, $k = 12$) has no 1-wide ribbons while the bottom ($n = 45$, $k = 14$) has three ribbons.

**Two types of ribbons.** There are two types of ribbons: ones that have ends, and ones that do not have ends. The bottom image shows 1-vertex wide versions of both types of ribbons with colored strips.

**With ends:** The fold at 22-23 of the **blue V** is a ribbon with ends (at 7-8 and 37-38). This ribbon is a 6-vertex loop (see *explainer* 11.6).

**Without ends:** There are two 1-wide ribbons that are without ends. *Such ribbons are created from pairs of parallel loops.* One is created from a pair of 6-vertex loops including vertices 1 and 2 (**green**) and the other from another pair of 6-vertex loops including vertices 4 and 5 (**red**).

**Counting vertices.** Note that none of the 30 vertices (five 6-vertex loops) used to create these three ribbons is a multiple of 3. The remaining 15 vertices ($n$ – 30) are multiples of 3. There are 7 paired vertices of the form $3j$ to $45 - 3j$, for $j = 1, \ldots, 7$, each creating one of the seven horizontal lines in the image. The final vertex is the identity vertex at the top of the image.

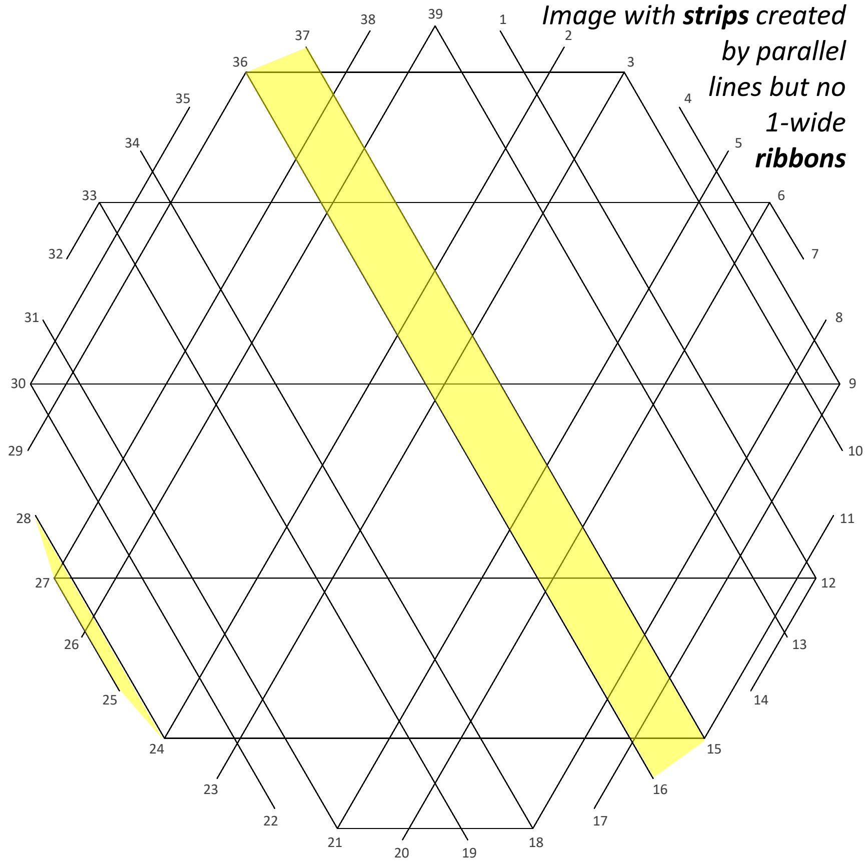


*Image with **strips** created by parallel lines but no 1-wide **ribbons***

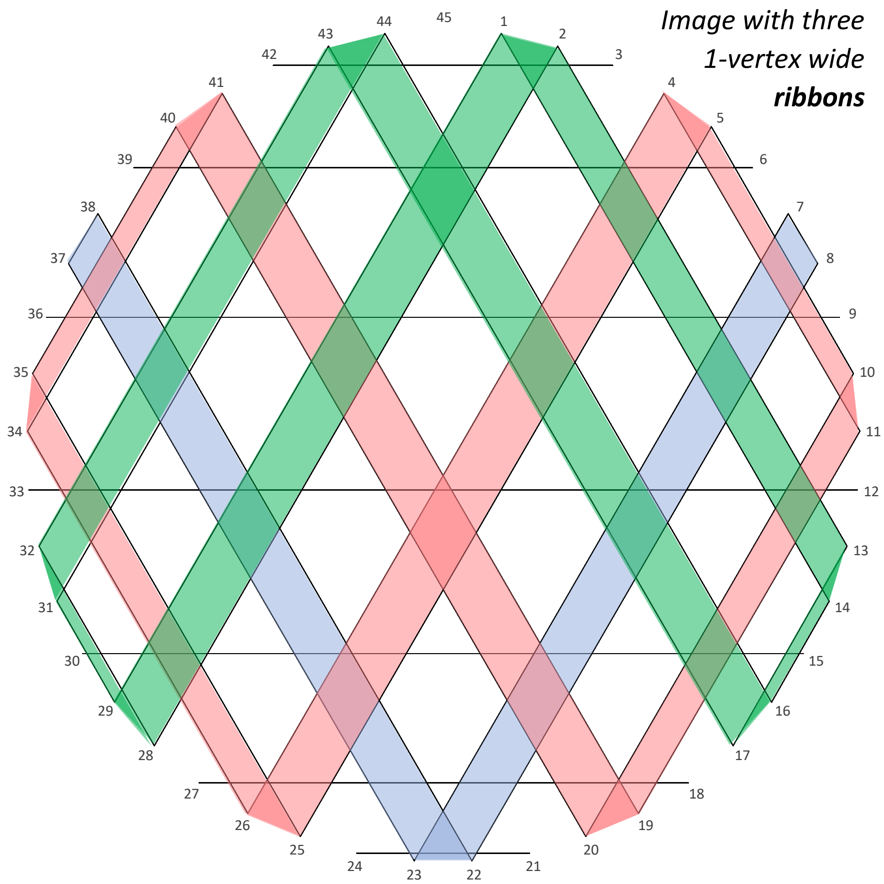


*Image with three 1-vertex wide **ribbons***