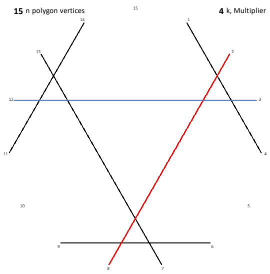
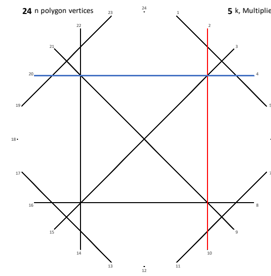
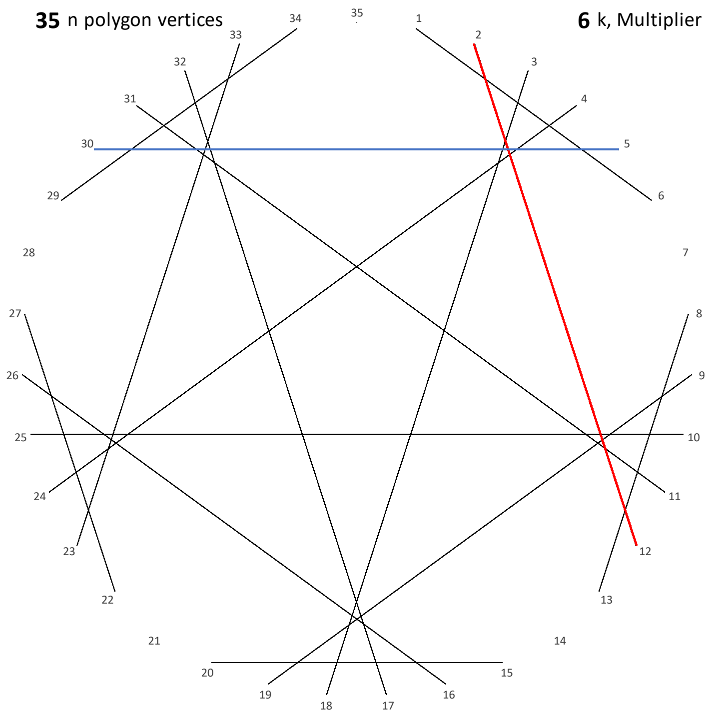


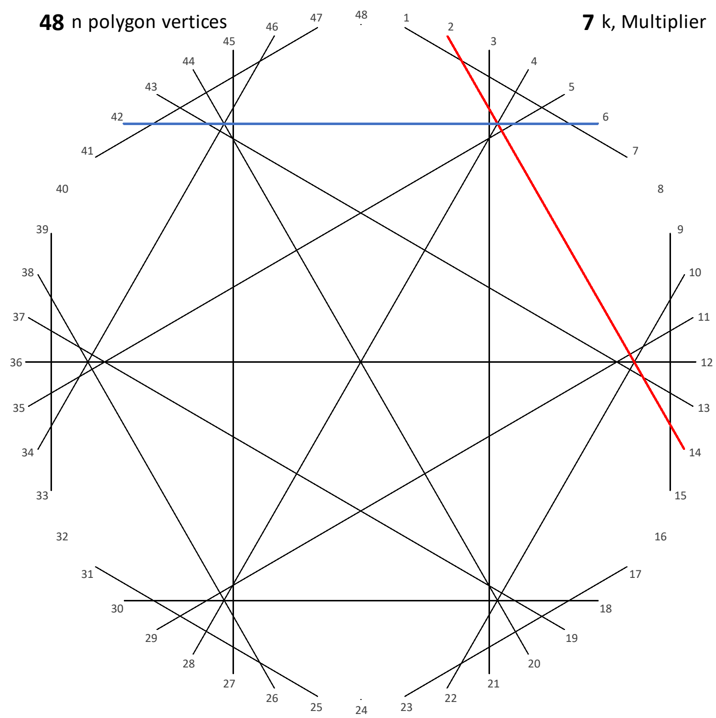
Centered Regular Polygons and Stars (CRPS) I

When $\boldsymbol{n} = \boldsymbol{k}^2 - 1$ the resulting image has centered regular ***k*-1**-gons (*CRP*) and centered regular stars (*CRS*). The ***k*** = 4 and 5 images show a regular centered triangle and square but when ***k*** increases beyond that, ***k*-1**-stars appear as well. Each image has ***k*** – 1 degrees of rotational symmetry and ***k***-1 identity vertices. The remaining $\boldsymbol{k}^2 - \boldsymbol{k}$ vertices are paired vertices so that in all there are ***k***·(***k***-1)/2 lines in each image.



Paired vertices allow us to describe a line by reference to a single vertex, and rotational symmetry means we only need concern ourselves with the portion of the image from 0 to ***k***+1. The intersection of the **blue horizontal *k*-1 line** and the **red 2 line** forms one corner of the largest ***k*-1**-gon in each image. The exterior angle to the ***k*-1**-gon created by these two lines is 2·(***k***+1)·180/***n***° = 360/(***k***-1)° since $\boldsymbol{k}^2 - 1 = (\boldsymbol{k}+1)\cdot(\boldsymbol{k}+1)$. The sum of exterior angles in the ***k*-1**-gon is thus 360°.



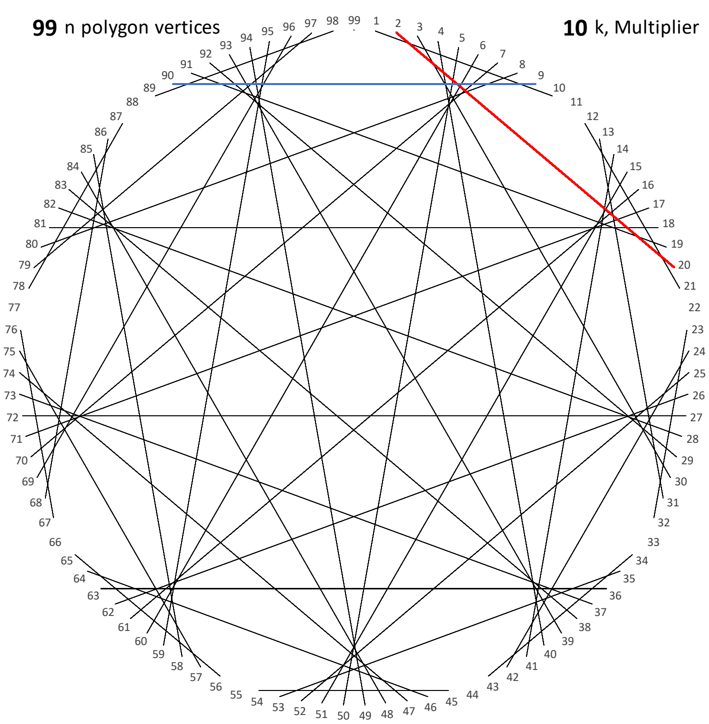


The focus here is on *centered* *regular* polygons and stars. The 3 small triangles for ***k*** = 4 are NOT *centered*. And one can readily envision *irregular* ***k***-2-stars in the ***k*** = 7 image above using internal pentagons from two of the internal hexagonal pie-pieces in the center of the image.

The CRPS use the vertices of the ***k***-gon. Each ***k*-1**-gon or ***k*-1**-star vertex is on the interior of the line segments created by the image. They are smaller than the regular polygons and stars discussed in **PARTS I and II** of **PwP**.

Increasing lines by 1 from 2 and decreasing by 1 from ***k***-1 creates both stars and smaller polygons at points of star line intersections. One can readily see the smaller pointed pentagon and hexagon in the ***k*** = 6 and 7 images above.

There is only one CRS when ***k*** = 6 and 7 as we see above. When ***k*** > 7 there are multiple centered ***k*-1**-stars. In each instance, the polygons created will alternate between being flat and pointed at the top as seen on p 2.



The image below reproduces the ***n*** = 99, ***k*** = 10 image shown at the bottom of page 1. The **dashed purple rays** show the first 9^{th} of the image (recall, these images have rotational symmetry of degree ***k***-1 (or 9 here). All other 9^{th}s are identical to this one. Further annotation of this 9^{th} is included via dots at vertices and intersection points in three different colors. The four 9-gons alternate with point at bottom and top and are noted by circled points in **black** numbered from 1 to 4.

The colored dots on vertices 3 through 8 highlight the symmetrically chosen vertices the same distance from the end of vertices 1 and 10. For example, the **blue dots at 3 and 8** are associated with the lines at 3 and 8 which create a **100° angle, 9-vertex, 2-jump CRS** (this is discussed in *explainer* 1.5a). Similarly, the **red dots are 60° angle 9-vertex 3-jump CRS** (such stars are cannot be drawn as a *continuously drawn star* as discussed in *explainer* 1.2 because GCD(3, 9) = 3). Finally, the **yellow dots** are associated with the **20° angle, 9-vertex, 4-jump CRS**.

Note that there is only **one 4-jump CRS** but there are **two 3-jump CRSs** (the smaller based on the lines used in creating the 4-jump CRS) and **three nested 2-jump CRSs** (the smaller based on the lines used in creating the 3- and 4-jump CRSs).