11. Counting Centered Regular Polygons and Stars II

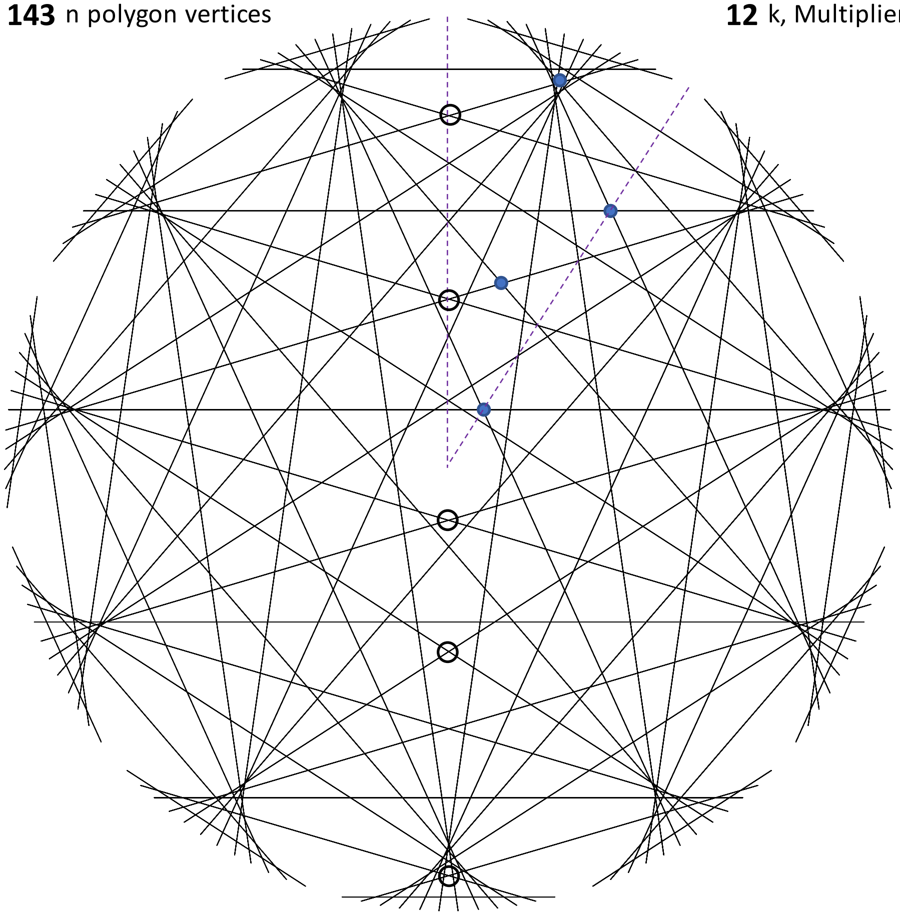
When ***n*** = ***k***^{2} – 1 the resulting image has centered regular ***k-1***-gons and ***k-1***-stars (CRPS). In **CRPS I** we saw that centered ***k-1***-gons (CRPs) exist when ***k*** ≥ 4 and centered ***k-1***-stars (CRSs) exist when ***k*** ≥ 6. We also noted that we can talk about lines by referring to vertices since each line is based on two paired vertices.

This explainer uses methods from **PART II of PwP** to count stars created in this situation and it is built on the foundation discussed on the second page of **CRPS I**.

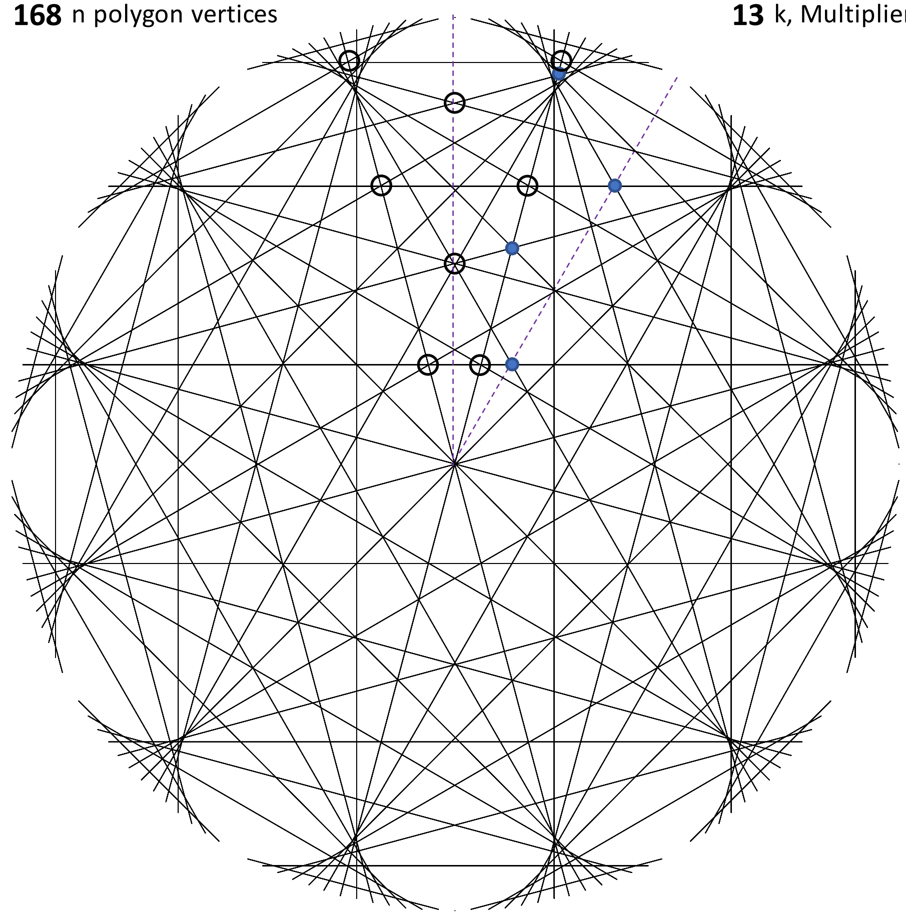
We saw in **CRPS I** that ***k*** = 6 and 7 have two CRPs and one CRS, but that with ***k*** = 10 there were 4 CRPs and 6 CRSs. Had we examined ***k*** = 11 we have the same number of CRPS as ***k*** = 10; the only difference would be that the ***k*** = 11 version has 10-gons and 10-stars (or 10-grams) and lines through the center of the circle (the centerline is located at “vertex” (***k***+1)/2 whether the line is visible or not). The visible centerlines when ***k*** is odd do not create new CRPs or CRSs.

The ***k*** = 12 and 13 images below create 11- and 12-gons and 11- and 12-grams. The five symmetric lines on each side of the cusp centerline create 5 CRPs (○) and a number of CRSs (lines 2-6 are to the right of center with lines 7-11 for ***k*** = 12 and lines 8-12 for ***k*** = 13 extend to the left of center). There is **1** 5-jump CRS, **2** 4-jump CRSs, **3** 3-jump CRSs and **4 2-jump CRSs**. The first explainer showed dots for all jump types in the first 9th, here only the **4 2-jump CRSs** are shown in **blue**.

143 n polygon vertices **12** k, Multiplier



168 n polygon vertices **13** k, Multiplier



The first **blue dot** is at the intersection of lines 3 and $\boldsymbol{k}$-2 and every other one alternates between edge (shown by dashed purple lines at vertices $\boldsymbol{n}$ and $\boldsymbol{k}$+1) and cusp centerline. Notice two things: there is now one more **blue dot** than in the $\boldsymbol{k}$ = 10 image on page 2 of **CRPS I**, and both images have the same number and “location” of dots. Had we colored in other dots, we would see one more dot for each color and a new color added for the lone 5-jump 11- or 12-point CRS.

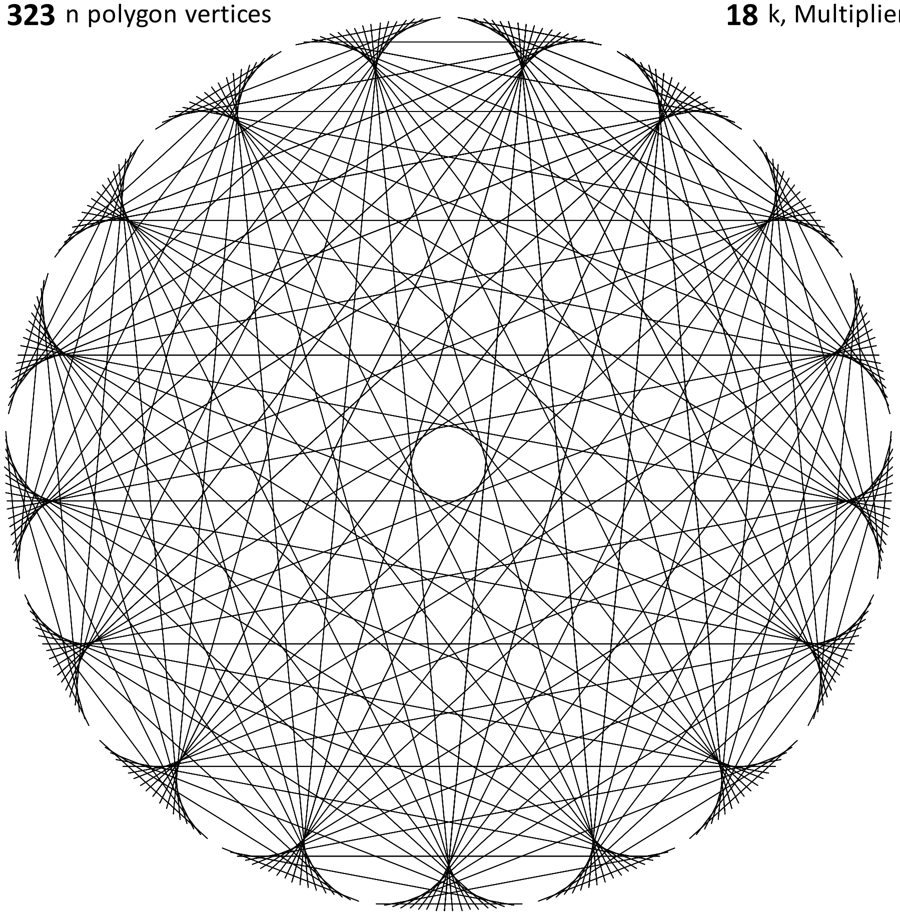
Counting. We have the following sequence of CRPs and CRSs. The number of CRPs increases by 1 for every increase of 2 in $\boldsymbol{k}$ starting at $\boldsymbol{k}$ = 4. The number of CRSs increases from 1 to 3 to 6 to 10 for every increase in $\boldsymbol{k}$ of two, starting at $\boldsymbol{k}$ = 6. *When the number of polygons increases by 1, the number of stars increases by the previous number of polygons.* The number of CRSs increases from 0 to 1 as CRP increases from 1 to 2; CRS increases by 2 (1+2 = 3) as CRP goes from 2 to 3; and so forth. The third of the three counting equations discussed in File 6 was the sum of the first $\boldsymbol{m}$ numbers, S($\boldsymbol{m}$). That equation is: $S(\boldsymbol{m}) = \boldsymbol{m}\cdot(\boldsymbol{m}+1)/2$.

The equation for the number of CRPs as a function of $\boldsymbol{k}$, $N_{CRP}(\boldsymbol{k})$, is: $N_{CRP}(\boldsymbol{k}) = INT(\boldsymbol{k}/2) - 1$ where INT is the integer function.

The equation for the number of CRSs as a function of $\boldsymbol{k}$, $N_{CRS}(\boldsymbol{k})$, is: $N_{CRS}(\boldsymbol{k}) = (N_{CRP}(\boldsymbol{k}) - 1)\cdot N_{CRP}(\boldsymbol{k})/2$.

An example. One can readily see 8 lines on each side of the cusp centerline for both images below (by looking at the lines creating the “first” cusp) as we have done above (the 9th is the line from 1 to $\boldsymbol{k}$ and is perpendicular to the centerline). This means that both images have $N_{CRP}(\boldsymbol{k})$ = 8 and $N_{CRS}(\boldsymbol{k})$ = 7·8/2 = 28.

323 n polygon vertices **18** k, Multiplier



360 n polygon vertices **19** k, Multiplier