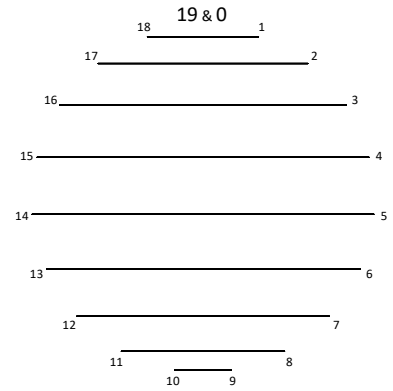


Using Excel to Explore Ribbon Structures



To understand the ribbon structure of the $n = (k+1)^2$ with k even image, it is worth noting that there are three types of vertices: the top vertex is an identity vertex; there are $k/2$ paired vertices (comprising k vertices); there are $k/2$ ribbons that are each $2 \cdot (k+1)$ -vertices long $(1 + k + k/2 \cdot 2 \cdot (k+1) = (k+1)^2)$.

To understand the ribbons, you must first focus on the $k/2 = 9$ paired vertices producing horizontal lines shown to the right. These lines are at multiples of $k+1 = 19$ (x in $19x$ are these multiples) and the sum of the two vertices is always $n = (k+1)^2$.

These horizontal lines ARE NOT at the same point in each of the $k-1$ cusps, but the vertices of the $k/2$ loops are symmetrically organized about these horizontal lines as shown in *361_18LoopExplainer.xlsx*.

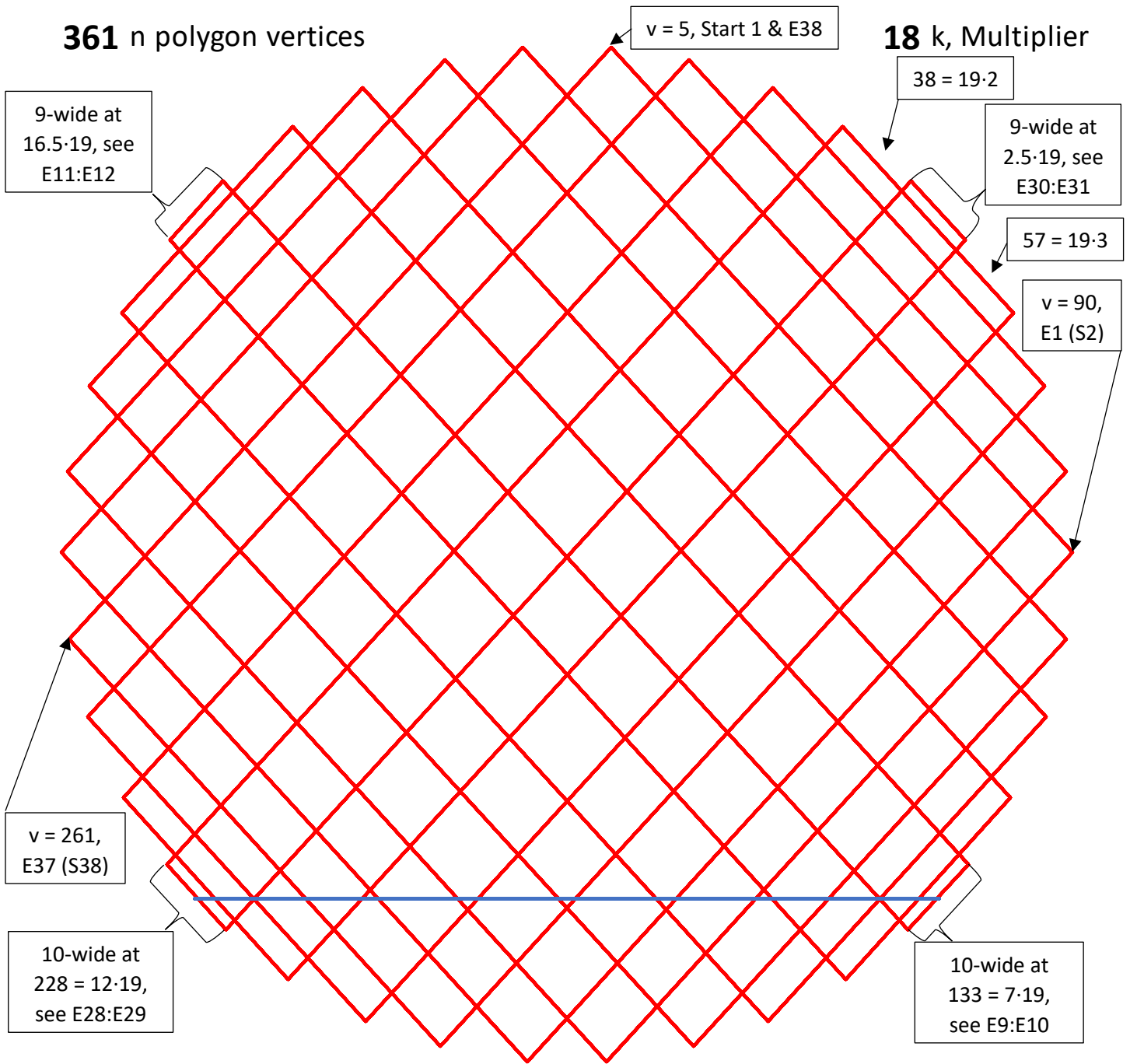
* Eq. below for Col. A-F.	v	$\Delta 19x$			
Vertex $v \text{ MOD } 19$	Line Ending	v near $19x$	Δv	lagged by 2	
5	5	start 1	0		
90	14	End1, S2	5	85	
176	5	E2 (S3)	9	86	9
280	14	E3 (S4)	15	104	10
347	5	E4 (S5)	18	67	9
109	14	E5 (S6)	6	-238	-9
157	5	E6 (S7)	8	48	-10
299	14	E7 (S8)	16	142	10
328	5	E8 (S9)	17	29	9
128	14	E9 (S10)	7	-200	-9
138	5	E10 (S11)	7	10	-10
318	14	E11 (S12)	17	180	10
309	5	E12 (S13)	16	-9	9
147	14	E13 (S14)	8	-162	-9
119	5	E14 (S15)	6	-28	-10
337	14	E15 (S16)	18	218	10
290	5	E16 (S17)	15	-47	9
166	14	E17 (S18)	9	-124	-9
100	5	E18 (S19)	5	-66	-10
356	14	E19 (S20)	19	256	10
271	5	E20 (S21)	14	-85	9
185	14	E21 (S22)	10	-86	-9
81	5	E22 (S23)	4	-104	-10
14	14	E23 (S24)	1	-67	-9
252	5	E24 (S25)	13	238	9
204	14	E25 (S26)	11	-48	10
62	5	E26 (S27)	3	-142	-10
33	14	E27 (S28)	2	-29	-9
233	5	E28 (S29)	12	200	9
223	14	E29 (S30)	12	-10	10
43	5	E30 (S31)	2	-180	-10
52	14	E31 (S32)	3	9	-9
214	5	E32 (S33)	11	162	9
242	14	E33 (S34)	13	28	10
24	5	E34 (S35)	1	-218	-10
71	14	E35 (S36)	4	47	-9
195	5	E36 (S37)	10	124	9
261	14	E37 (S38)	14	66	10
5	5	E38 (S39)	0	-256	-10

By placing a number between 1 and 9 the crème-colored cell A3 we see what happens. The left table shows the “cross-over” value ($v = 5$) with smallest difference between narrow (9) and wide (10) way of viewing the ribbon structure as all 4 ends are readily visible in the image on the next page which suppresses the cardioid. These are highlighted in red in the Δv column (Excel column E automatically highlights these cells). The right table below provides summary information for all v from 1 to 9.

1. The most obvious feature is in the second column on the left where v and $k+1 - v$ alternate (5, 14, 5, 14, ...). This is most easily understood by noting that multiplying by k is the same as multiplying by $-1 \text{ MOD } k+1$. $a = v \text{ MOD } 19$ and $k = 18 = -1 \text{ MOD } 19$ then $a \cdot k = -1 \cdot v \text{ MOD } 19$ is an example of modular multiplication. In our context, $19 - v = -v \text{ MOD } 19$.
2. Two of the ends, highlighted in yellow are at E10 and E29 and the size of the end is $2v$, $v = 1 - 9$, because these two vertices are equidistant from the same $19x$ (7 for E9:E10 and $19 - 7 = 12$ for E28:E29). This is the horizontal $19x$ blue line with paired vertices $133 = 19 \cdot 7$ and $228 = 19 \cdot 12$.
3. The “ v near $19x$ ” values are symmetric about E9:E10 and E28:E29 but switch between v and $-v$ (compare the line ending at E9 with E11, E8 with E12, etc., and note that E19’s 19 coincides with 0). Lines 1 to 9 are reversed by lines 11 to 19. This ensures parallel lines in the image.
4. The odd-width ribbons span two $19x$ vertices and are noted using 0.5 values (such as 2.5 for E30:E31 or $19 - 2.5 = 16.5$ for E11:E12 in the example to the left). Viewed this way, the ribbon is 9-vertices wide. Generally, we use the smaller of the two widths as it is easier to see.
5. All (smallest-sized) ribbons from 1-9 are possible in this situation.

Even values occur up to the cross-over value with odd values thereafter.	End of first line for:		Ribbon size (and general $ \Delta 19x $)	Smallest near 19x vertex	Second smallest near 19x vertex
	Smallest v	smallest Δv			
1	10	1	2	9	0.5
2	10	15	4	1	8.5
3	10	7	6	8	1.5
4	10	3	8	2	7.5
5	12	10	9	2.5	7
6	18	10	7	6.5	3
7	6	10	5	3.5	6
8	16	10	3	5.5	4
9	9	10	1	4.5	5

*Eq. in A4=MOD(A3*18,361) then drag to A41.
 Eq. in B3=MOD(A3,19) then drag to B41.
 Eq. in D3=ROUND(A3/19,0) then drag to D41.
 Eq. in E4=A4-A3 then drag to E41.
 Eq. in F5=D5-D3 then drag to F41.



Given $v = 5$, both 9-wide and 10-wide ribbons are readily visible. The blue paired vertices at 133 to 228 are the midpoint of the 10-wide strip ends. (If $v \neq 5$, the wider version may be harder to see but, the sum of widths remains 19.)

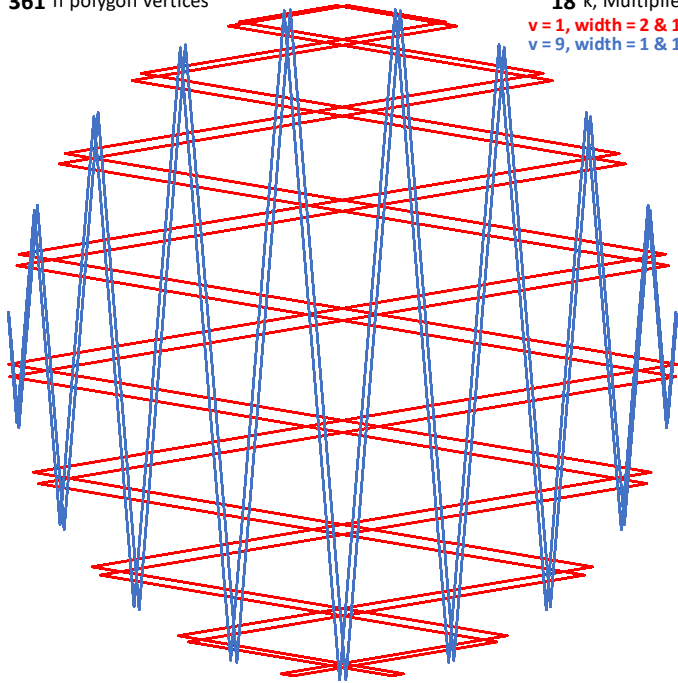
By contrast, the 9-wide ends have different nearest $19x$ vertices. On the upper right, the annotated 9-wide end is from 43 to 52. The 43 vertex is closer to the $2 \cdot 19 = 38$ horizontal line but the 52 vertex is closer to the $3 \cdot 19 = 57$ horizontal line. This is the reason this end is described as a 9-wide ending at $2.5 \cdot 19$.

Additionally, the first and last lines in the image are noted so that you can follow the loop, line-by-line, should you wish to do so.

It is worth noting that the vertices alternate 9, 10, 9, 10, ... around the edge. You can think of this structure as either an open-ended 9-vertex wide ribbon, or as an open-ended 10-vertex wide ribbon.

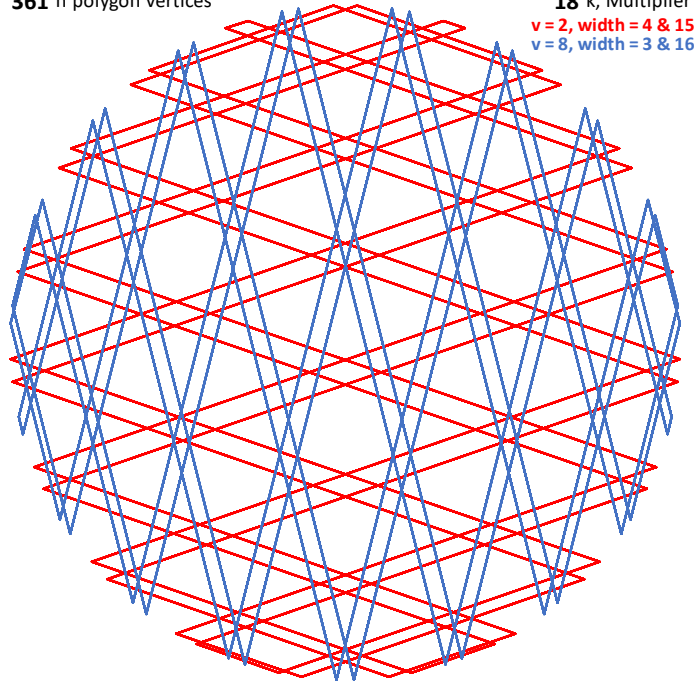
The four images on the next page show the ribbon structure for the other eight smallest starting vertices. These are shown two at a time from narrowest to widest (when viewed using the smaller ribbon size in each instance).

361 n polygon vertices



18 k, Multiplier
 $v = 1$, width = 2 & 17
 $v = 9$, width = 1 & 18

361 n polygon vertices



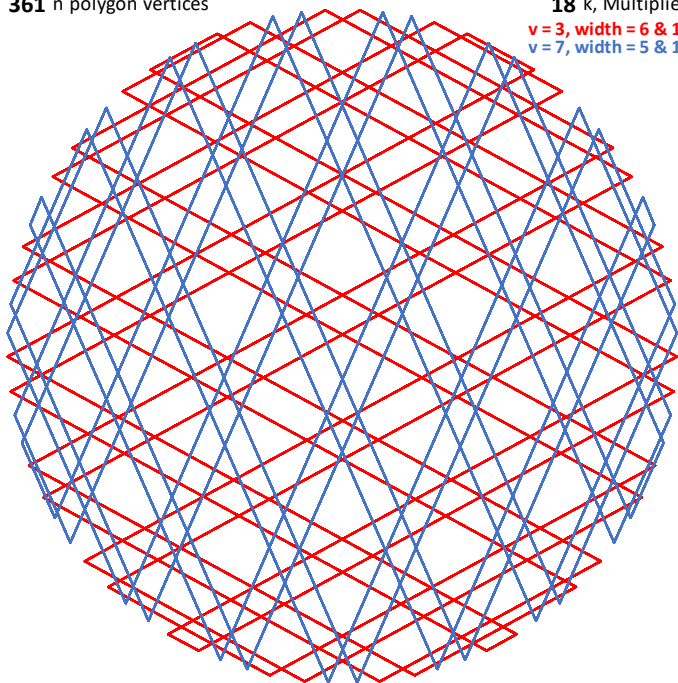
18 k, Multiplier
 $v = 2$, width = 4 & 15
 $v = 8$, width = 3 & 16

In each panel, the blue image has a ribbon that is one vertex smaller than the red image. The upper left shows 1-blue and 2-red. The easiest way to compare the widths of red and blue ribbons is to look at bottom or top for blue and left and right for red since this is when the parallel lines appear farther apart.

Consider for a moment each color on its own. The blue image in the upper left has no internal blue intersections (aside from the normal cross-over of lines near the fold (see *explainer* 11.6c) but the other seven individual ribbons have one or more. A quick check will show that the number of internal intersections is one less than the width of the ribbon. (If the other color introduces too much “noise” in the image, use the Excel file and the toggles in N3:O3 to show only one color at a time.

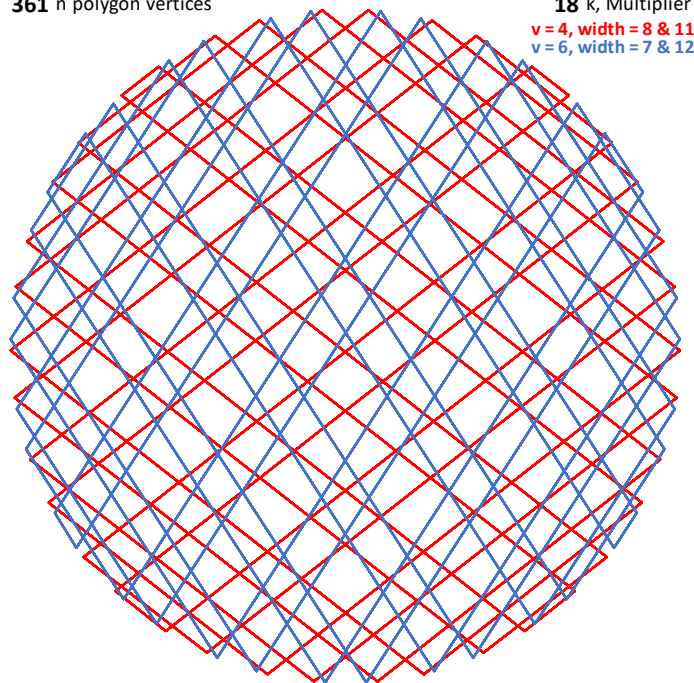
Interestingly, the ribbon widths also provide angular measures of the single-color parallelograms in each image. As we have seen, starting vertex v has ribbon widths of $2v$ and $19 - 2v$. The angles are $180 \cdot 2v/19$ and $180 \cdot (19 - 2v)/19$. To calculate cross-color parallelogram angles, follow the rules for calculating interior angles discussed in *explainer* 11.3a.

361 n polygon vertices



18 k, Multiplier
 $v = 3$, width = 6 & 13
 $v = 7$, width = 5 & 14

361 n polygon vertices



18 k, Multiplier
 $v = 4$, width = 8 & 11
 $v = 6$, width = 7 & 12