# 11. Deconstructing a Clockface using the Cardioids with Loops file (*Note:* This is a 5-page explainer)

The image to the right could easily be used as a clockface. There are empty spaces on the hour, major cusps on the half-hour and minor cusps on the quarter hour.

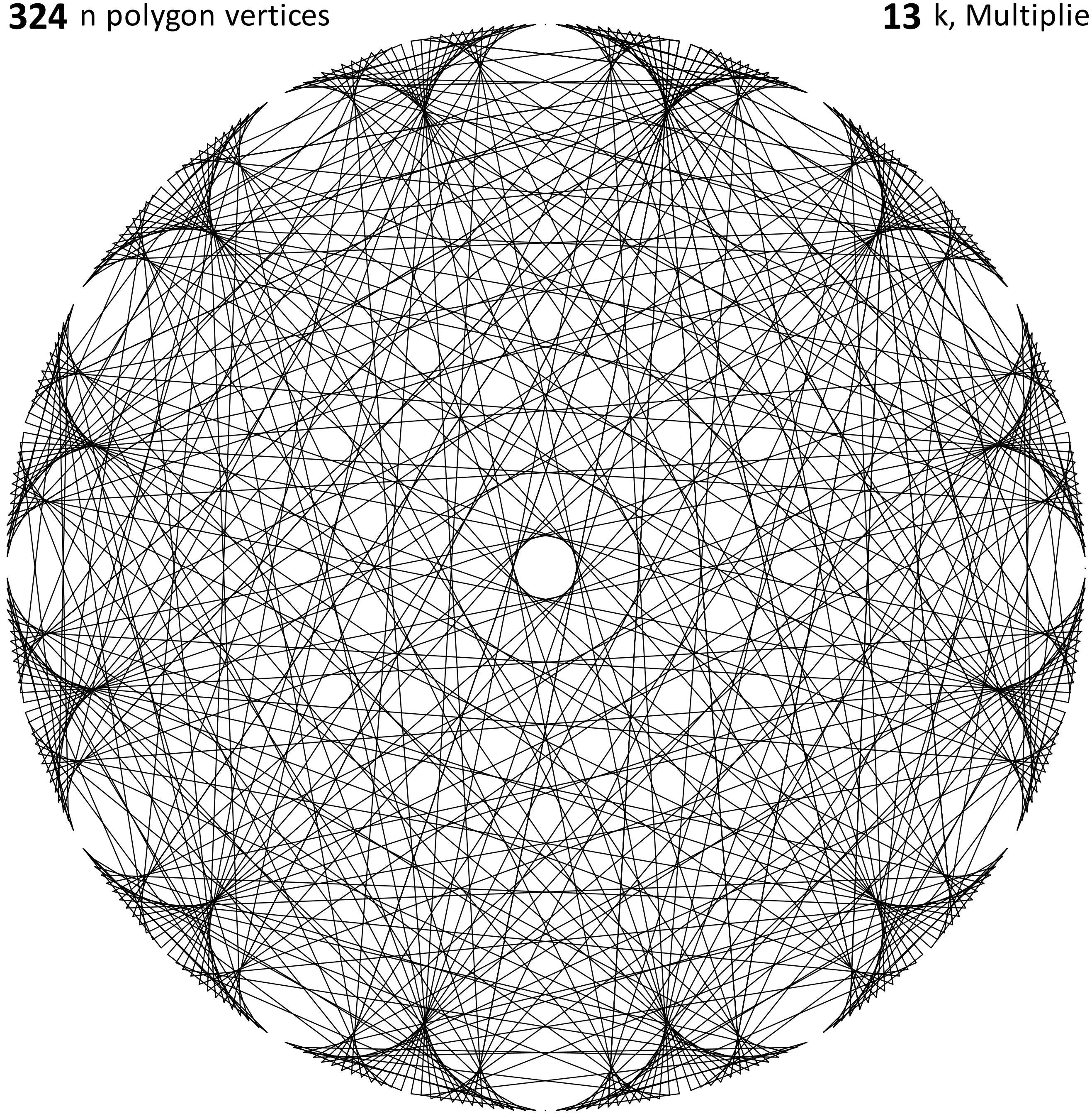
All vertices with a line have two lines, one in and one out. Each large cusp is created from 26 lines GOING OUT from the vertices spanning the cusp. A careful inspection reveals that each minor cusp on either side of the major cusp is 13 vertices wide and is formed from lines COMING INTO the vertices creating half the large cusp

**Each of the 324 vertices is part of one of four types of loops.** The empty spaces are created by 12 identity vertices at $\boldsymbol{v} = 27 \cdot \boldsymbol{k}$ with $1 \leq \boldsymbol{k} \leq 12$. There are 8 loops each of the other three loop types: each will be highlighted on a separate page. The table below shows the smallest vertex in each of these loops and notes the pairs highlighted (A & E). These pairs are chosen to form a regular hexagram (6-vertex, 2-jump star) by overlapping two equilateral triangles.

| Loop label | Smallest vertex in loop | | |
|---|---|---|---|
| **A** | **1** | **3** | **9** |
| **B** | 2 | 6 | 18 |
| **C** | 4 | 12 | 36 |
| **D** | 5 | 15 | 45 |
| **E** | **7** | **21** | **63** |
| **F** | 8 | 24 | 72 |
| **G** | 10 | 30 | 90 |
| **H** | 11 | 33 | 99 |
| | **Length of loop** | | |
| | **27** | **9** | **3** |
| **See page:** | **2** | **3** | **4** |

**324** n polygon vertices

**13** k, Multiplier



2. This page shows two 27-vertex loops. Note the **small half blue**, **half red** hexagonal star near the center of the image. The loops are shown in the table to the left. It is readily apparent that there are two parallel lines in each of the three directions.

Consider the two lines: **E1 to E2 - Red**, & **E26 to E27 - Blue**.

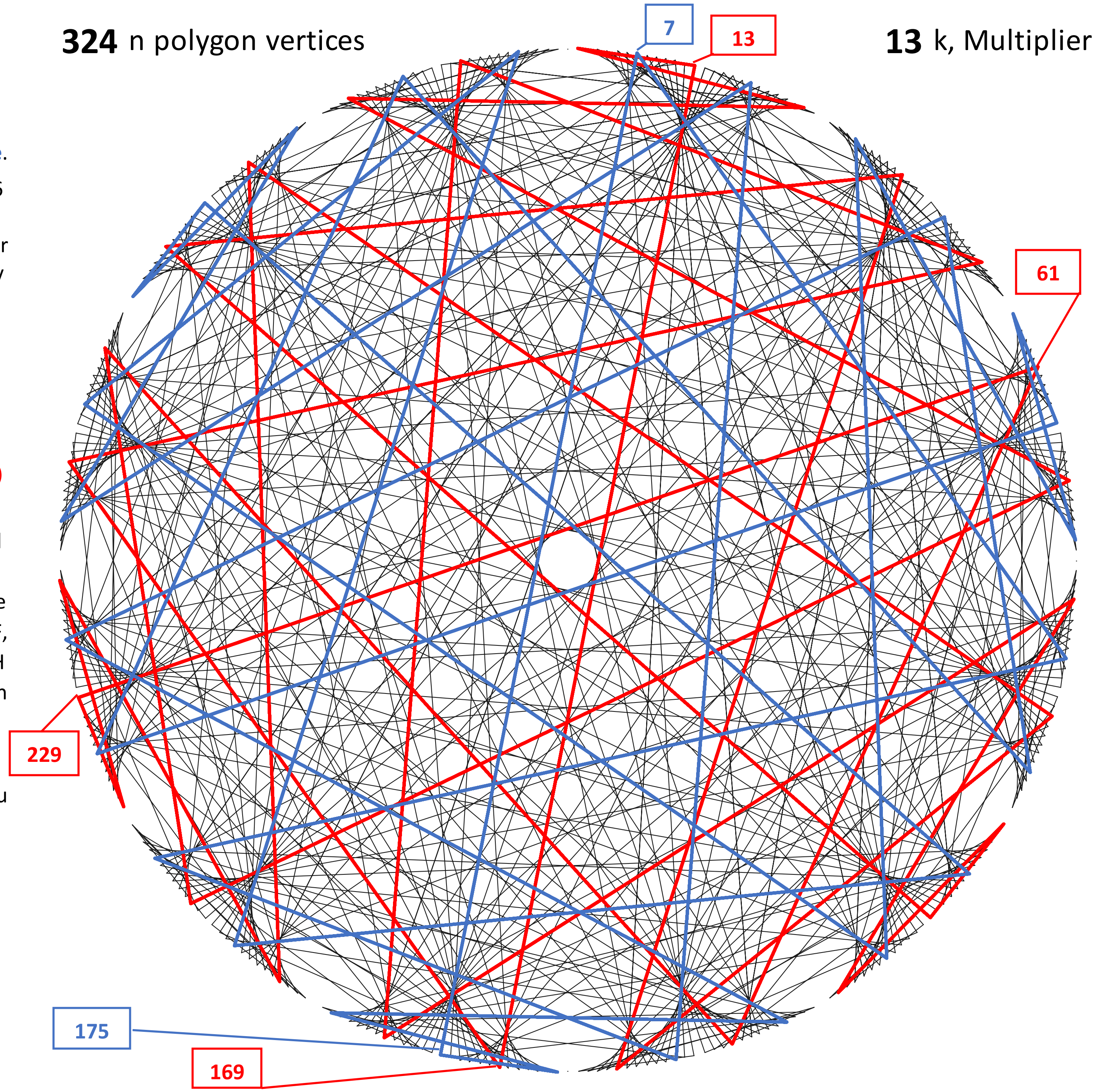
| ☑ | ☑ | blue overlays red |
|---|---|---|
| A | B | |
| 1 | 7 | ***Start* 1** |
| 13 | 91 | ***End* 1, S2** |
| 169 | 211 | **E2 (S3)** |
| 253 | 151 | **E3 (S4)** |
| 49 | 19 | **E4 (S5)** |
| 313 | 247 | **E5 (S6)** |
| 181 | 295 | **E6 (S7)** |
| 85 | 271 | **E7 (S8)** |
| 133 | 283 | **E8 (S9)** |
| 109 | 115 | **E9 (S10)** |
| 121 | 199 | **E10 (S11)** |
| 277 | 319 | **E11 (S12)** |
| 37 | 259 | **E12 (S13)** |
| 157 | 127 | **E13 (S14)** |
| 97 | 31 | **E14 (S15)** |
| 289 | 79 | **E15 (S16)** |
| 193 | 55 | **E16 (S17)** |
| 241 | 67 | **E17 (S18)** |
| 217 | 223 | **E18 (S19)** |
| 229 | 307 | **E19 (S20)** |
| 61 | 103 | **E20 (S21)** |
| 145 | 43 | **E21 (S22)** |
| 265 | 235 | **E22 (S23)** |
| 205 | 139 | **E23 (S24)** |
| 73 | 187 | **E24 (S25)** |
| 301 | 163 | **E25 (S26)** |
| 25 | 175 | **E26 (S27)** |
| 1 | 7 | **E27 (S28)** |

These lines are 6 vertices apart from one another … therefore they are parallel:
**Red: 13 to 169**
**Blue: 175 to 7.**

Interior angles are 60°.
For example:
**(61-13)+(229-169) = 108** or 324/3.

Three additional small hexagonal stars are possible using loops B & F, C & G, and D & H from the table on the previous page.

Alternatively, you can create two small 12-point regular stars using loops A, C, E, and G for one, and B, D, F, and H for the other.



3. This page shows two 9-vertex loops. Note the **medium** **half blue**, **half red** hexagonal star near the center of the image. The loops are shown on the left below. It is readily apparent that there are two parallel lines in each of the three directions.

Consider the two lines: **E1 to E2 for Red**, & **E8 to E9 for Blue**. These lines are 18 vertices apart from one another … therefore they are parallel:

**Red: 39 to 183** **Blue: 201 to 21**

Once again, we see interior angles are 60°. For example: **(75 - 39) + (255 - 183) = 108** or 324/3.

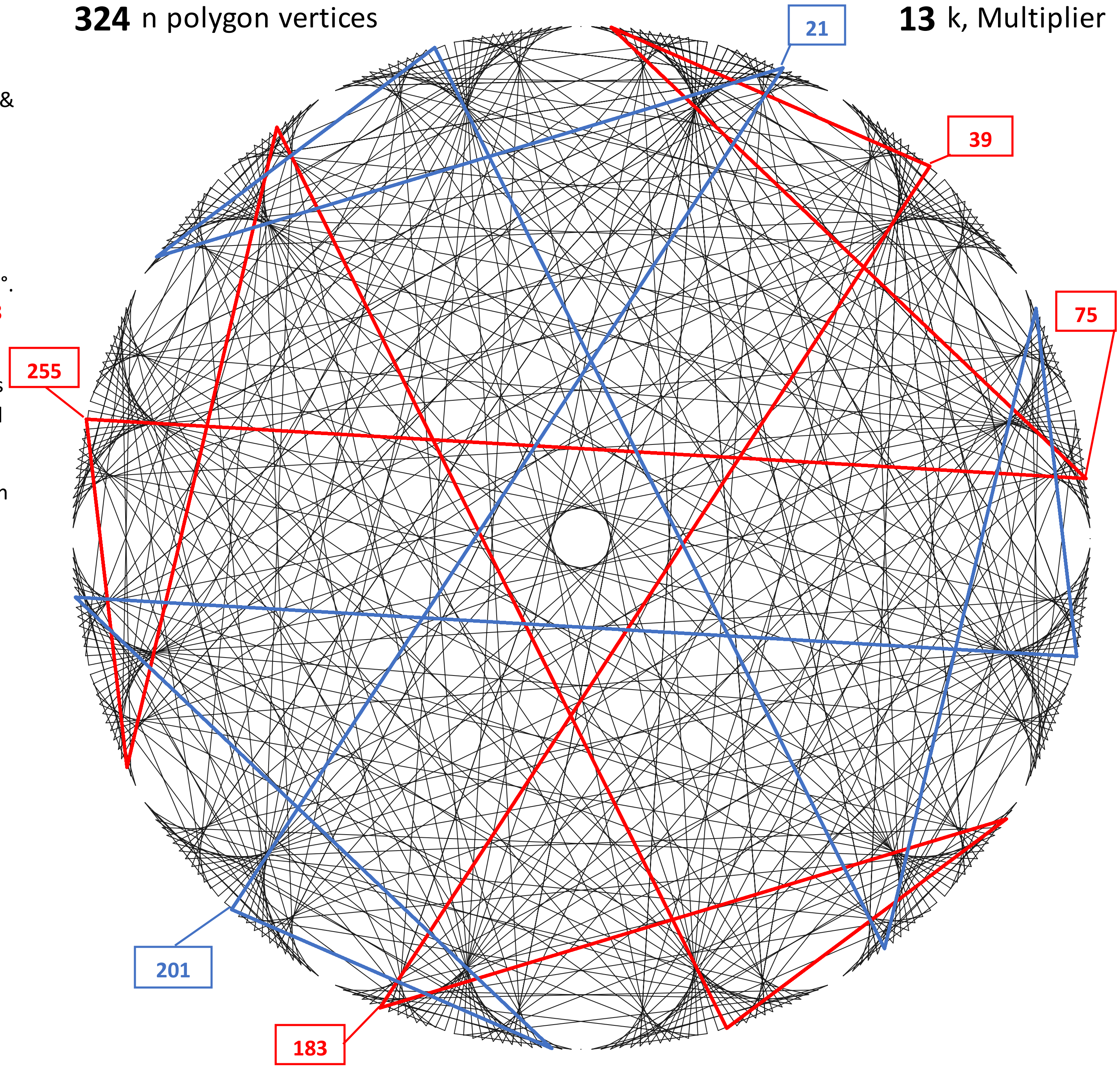
Three additional medium hexagonal stars are possible using loops B & F, C & G, and D & H from the table on the first page.

Alternatively, you can create two medium 12-point regular stars using loops A, C, E, and G for one, and B, D, F, and H for the other.

| ☑ | ☑ | blue overlays red |
|---|---|---|
| A | B | |
| 3 | 21 | *Start* 1 |
| 39 | 273 | *End* 1, S2 |
| 183 | 309 | E2 (S3) |
| 111 | 129 | E3 (S4) |
| 147 | 57 | E4 (S5) |
| 291 | 93 | E5 (S6) |
| 219 | 237 | E6 (S7) |
| 255 | 165 | E7 (S8) |
| 75 | 201 | E8 (S9) |
| 3 | 21 | E9 (S10) |

**324** n polygon vertices

**13** k, Multiplier



4. This page shows two 3-vertex loops. Note the **Large** half blue, **half red** hexagonal star using vertices of the image. The loops are shown on the left below. It is readily apparent that there are two parallel lines in each of the three directions.

Consider the two lines: **E1 to E2 for Red**, & **E2 to E3 for Blue**. These lines are 54 vertices apart from one another … therefore they are parallel:

**Red: 117 to 225** **Blue: 279 to 63**

The inscribed angles are 60°.
For example: **225 - 117 = 108** or 324/3.

Additionally note: **63** – **9** = 54 = 324/6.

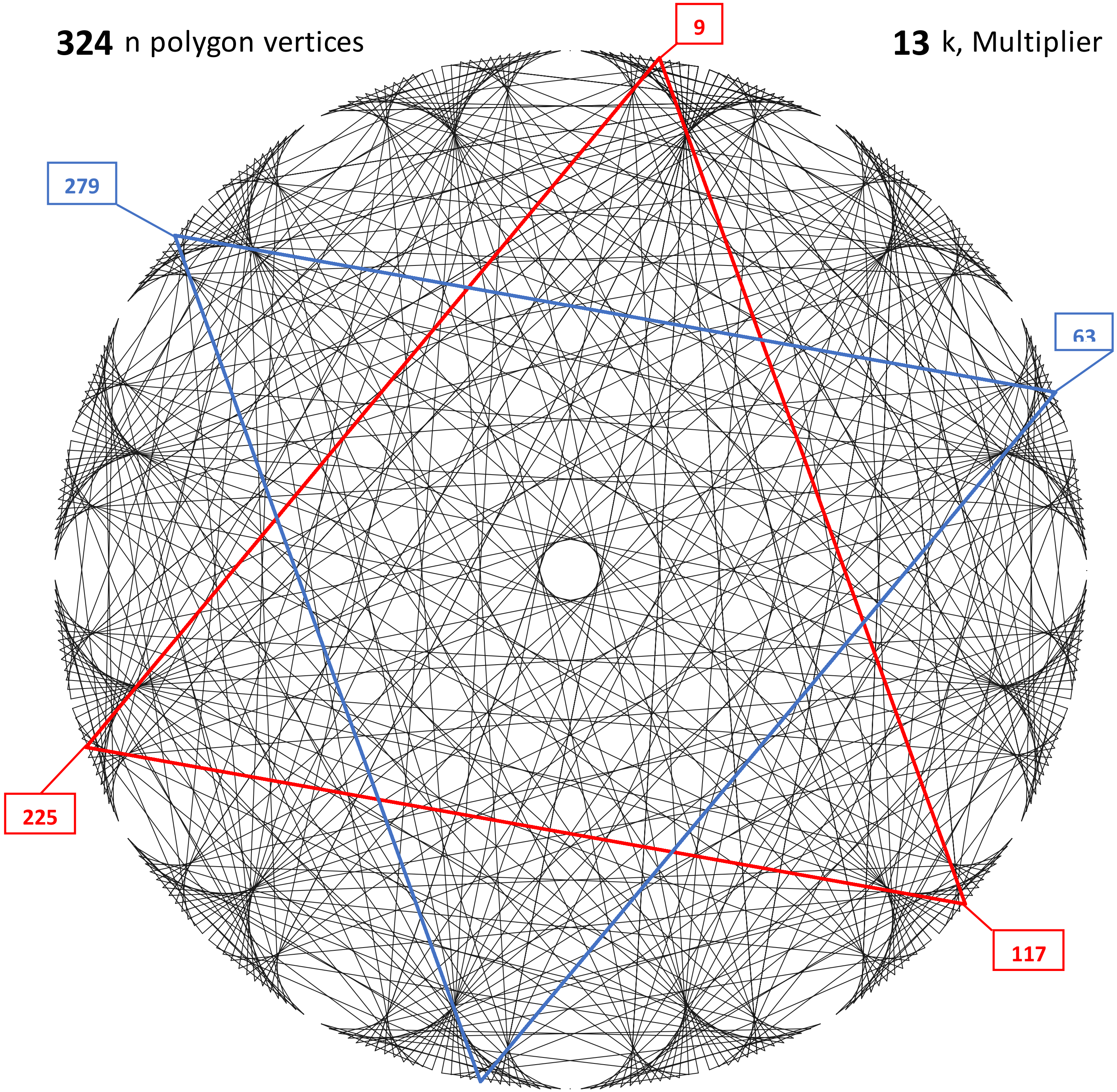
Three additional large hexagonal stars are possible using loops B & F, C & G, and D & H from the table on the first page.

Alternatively, you can create two large 12-point regular stars using loops A, C, E, and G for one, and B, D, F, and H for the other.

To wrap up: We have argued that the image is created from 4 different types of loops. There are 12 identity loops, and 8 loops each that are 27- 9- and 3-vertices long. These loops use all 324 vertices because:

$$324 = 12 + 8{\cdot}27 + 8{\cdot}9 + 8{\cdot}3.$$

| ☑ | ☑ | blue overlays red |
|---|---|---|
| A | B | |
| 9 | 63 | ***Start*** **1** |
| 117 | 171 | ***End*** **1, S2** |
| 225 | 279 | **E2 (S3)** |
| 9 | 63 | **E3 (S4)** |



# 5. **This page outlines the steps taken to create the above analysis.**

The loop overlays are critical in deconstructing an image. They led me to create the table on p. 1, reproduced here:

| Loop label | Smallest vertex in loop | | |
|---|---|---|---|
| **A** | **1** | **3** | **9** |
| **B** | 2 | 6 | 18 |
| **C** | 4 | 12 | 36 |
| **D** | 5 | 15 | 45 |
| **E** | **7** | **21** | **63** |
| **F** | 8 | 24 | 72 |
| **G** | 10 | 30 | 90 |
| **H** | 11 | 33 | 99 |
| | **Length of loop** | | |
| | **27** | **9** | **3** |
| **See page:** | **2** | **3** | **4** |

1. It is always useful to note the rotational symmetry (cell U10 = 12).

2. My initial focus was on the first cusp, so the size had to be determined. Given rotational symmetry, the cusp involves vertices 1 – 27 (27 = 324/12). A quick check of 27 shows that it is indeed an identity vertex (type 27 in N5).

3. Type numbers into cells N5 and O5 two at a time and note the results in the green area on the Excel sheet.

4. In this instance, it became clear that not all loops were the same size. 1 and 2 are both 27-vertex, but 3 is 9-vertex.

5. Once I got to vertex 9 it became clear that a third loop style was going to be part of the image, with 3-vertex loops.

6. Note that the first number NOT in the table is 13. This makes a ton of sense because ***k*** = 13 so 13 is part of loop 1.

   The vertex loop columns are formatted to automatically highlight the smallest number in the loop so typing 14 in N5 to see if 14 is the start of a loop yields the smallest number in the loop, 2, as E25. This means 14 is already done.

7. Note that all 27-vertex loops vertices are not divisible by 3. This means that all vertices in these loops is also not divisible by 3 because 324 IS divisible by 3 and all vertices are multiples of the initial vertex and powers of 13. The simplest example is the 27th line in the 1 loop (E26 to E27): $25 \cdot 13 = 325 = 324 + 1$.

   Two-thirds of the numbers between 1 and 324 are NOT multiples of three and $8 \cdot 27 = 216 = 2/3 \cdot 324$ so all vertices that that are not divisible by 3 are accounted for by these eight 27-vertex loops.

8. Note that all 9-vertex loops are multiples of 3 but not 9.

9. Note that all 3-vertex loops are multiples of 9.

10. If you look at all eight 27-vertex loops you will see that each loop "touches" each of the 12 major cusps. The same cannot be said of 9- and 3-loops (of course) since they can touch <u>at most</u> 9 and 3 cusps.

11. Consider the two 3-vertices loops with vertices touching the first cusp. Since they are equilateral triangles, they also touch the 5th and 9th cusps, each in the same location within the cusp as in the first cusp.

    This means that there will be additional equilateral triangles with smallest vertex at 9 and 18 vertices into cusps 2, 3, and 4 due to rotational symmetry.

    This means I expected 3-vertex loops at 27, 54, and 81 more than 9 and 18. These are the final six equilateral triangle 3-vertex loops.

12. At this point, I thought I had all vertices. I checked this way: 12 identity vertices + $8 \cdot 27 + 6 \cdot 9 + 8 \cdot 3 = 12 + 216 + 54 + 24 = 306$. This leaves 324 – 306 = 18 vertices unaccounted for.

13. The 18 remaining vertices must be divisible by 3 but not 9.

    Adding 27 to the first two numbers (3 and 6) and checking them in N5 and O5 produced the remaining two 9-vertex loops.

14. The image appeared to have multiple 6-vertex 2-jump regular stars (hexagrams) as well as 12-vertex 4-jump regular stars and this visual hunch was confirmed by the chosen loops **A** and **E** (the **first** and **fifth**) annotated in **red** and **blue** on pages 2 – 4.