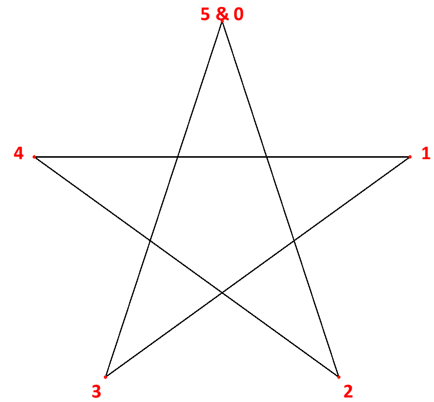
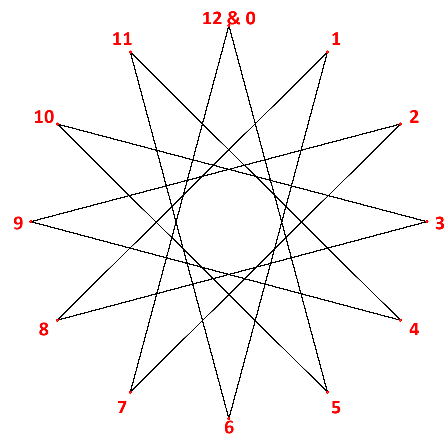
About **MOD**: When counting round and round, the **remainder** is all that matters

To create the simplest star, the $\boldsymbol{n} = 5$, $\boldsymbol{J} = 2$ pentagram using the **PwP** files, you start at the top then draw a line from 0 (the top) to the second vertex (in the lower right), then to the 4^{th} vertex, and so on. The rule is to add 2 and draw the next line. But the third line has an end that is 2 more than 4 or 6 … but there are only 5 vertices in the $\boldsymbol{n} = 5$ gon. It is natural to put the end of the third line at vertex 1 (since it is 1 after 5, the top of the 5-gon).



This same idea applies if it is now 11 o'clock and your friend says, *lets meet in 2 hours*. You know that is at 1 o'clock not 13 o'clock. In both instances, **all that matters is what is left over** after you pass around the top of the image (or go past 12 o'clock). The location of the next point is the ***remainder*** upon division by 12.

To push this a bit further, suppose we create a 12-point star with 5 jumps between used vertices. This is easy to think of in terms of hour jumps because 12 equally spaced vertices creates a clock face if we start at the top which we call 0 (or 12 & 0 instead of 12). The jumps and lines look like this:



Counting rule for creating a 12-point, 5-jump star: Add 5 starting at 0												
Line number L	1	2	3	4	5	6	7	8	9	10	11	12
*Ending number m	5	10	15	20	25	30	35	40	45	50	55	60
Ends at vertex **v**	**5**	**10**	**3**	**8**	**1**	**6**	**11**	**4**	**9**	**2**	**7**	**0**
Times past top t	0	0	1	1	2	2	2	3	3	4	4	5
Equations:* m = 5·L; **v = MOD(m,12); t = INT(m/12). *Note:* m = 12·t + **v**												

About MOD. The third and fourth rows were created using the MOD and INT function in Excel. the INT function provides the integer portion of a number so INT(12.37) = 12 and MOD is the remainder once one number is divided by another. MOD($\boldsymbol{b}$,$\boldsymbol{c}$) is Excel's function that produces a remainder $\boldsymbol{r}$ with $0 \le \boldsymbol{r} < \boldsymbol{c}$ once a number $\boldsymbol{b}$ is divided by a divisor, $\boldsymbol{c} > 1$. Thus, $\boldsymbol{r}$ = MOD($\boldsymbol{b}$,$\boldsymbol{c}$) if $\boldsymbol{b} = \boldsymbol{c}\cdot\boldsymbol{x} + \boldsymbol{r}$ where $\boldsymbol{x}$ is an integer and $0 \le \boldsymbol{r} < \boldsymbol{c}$. (Mathematicians write this as $\boldsymbol{b}$ mod $\boldsymbol{c} = \boldsymbol{r}$.)

If $\boldsymbol{b}$ is a multiple of $\boldsymbol{c}$, $\boldsymbol{r} = 0$, if not, $0 < \boldsymbol{r} < \boldsymbol{c}$. In the above table, the 12^{th} ending number, 60, is a multiple of 12.

The MOD and INT functions are related (as noted in the last line of the table): $\boldsymbol{b} - \boldsymbol{c}\cdot$INT($\boldsymbol{b}/\boldsymbol{c}$) = MOD($\boldsymbol{b}$,$\boldsymbol{c}$).

Why is MOD useful? The remainder function allows us to only worry about left-overs when doing additional calculations. One need not work with larger numbers when smaller numbers will suffice to answer the question. There are a number of properties of modular arithmetic (that you can read about on Wikipedia) but in this initial pass, I want to focus on two: addition and multiplication by a constant.

Addition by a constant. If $\boldsymbol{d}$ is an integer and $\boldsymbol{r}$ = MOD($\boldsymbol{b}$,$\boldsymbol{c}$) then $\boldsymbol{r}+\boldsymbol{d}$ = MOD($\boldsymbol{b}+\boldsymbol{d}$,$\boldsymbol{c}$).

For example, in the table above, since 2 = MOD(50,12) then 2+5 = 7 = MOD(50+5,12).

Multiplication by a constant. If $\boldsymbol{d}$ is an integer and $\boldsymbol{r}$ = MOD($\boldsymbol{b}$,$\boldsymbol{c}$) then $\boldsymbol{r}\cdot\boldsymbol{d}$ = MOD($\boldsymbol{b}\cdot\boldsymbol{d}$,$\boldsymbol{c}$).

In the table above, since 3 = MOD(15,12), 6 = MOD(15·2,12), 9 = MOD(15·3,12), and 12 = MOD(15·4,12) but note that 0 = MOD(12,12) which is why this last multiple is written as 0 = MOD(60,12).

An example from *explainer* 2.1b. The location of the $\boldsymbol{k}^{th}$ endpoint is easily obtained using the remainder function (MOD). Given $\boldsymbol{n}\cdot\boldsymbol{S} = 8$ possible endpoints, the $\boldsymbol{k}^{th}$ is located at $\boldsymbol{r}$ = MOD($\boldsymbol{k}\cdot\boldsymbol{P}$, $\boldsymbol{n}\cdot\boldsymbol{S}$) with $\boldsymbol{P} = 3$, so the 7^{th} is at 5 = MOD(7·3, 4·2) above. This is the midpoint between the 2^{nd} and 3^{rd} vertex because $\boldsymbol{r}/\boldsymbol{S} = 5/2 = 2.5$ in this instance and it was attained during the third time around the circle since $2 < (\boldsymbol{k}\cdot\boldsymbol{P})/(\boldsymbol{n}\cdot\boldsymbol{S}) = 21/8 < 3$.