Identifying prime numbers using the ancient *Sieve of Eratosthenes* (from the 3rd century BC)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300
301	302	303	304	305	306	307	308	309	310
311	312	313	314	315	316	317	318	319	320
321	322	323	324	325	326	327	328	329	330
331	332	333	334	335	336	337	338	339	340
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The Greek mathematician Eratosthenes proposed a simple mechanism for finding prime numbers. *Primes are numbers that cannot be obtained as the product of two smaller numbers.* If a number can be expressed that way, it is a *composite* number. The 8 panels above show the first 8 iterations of the sieve and the images below show the results: there are 73 primes less than 370 (370 was chosen because some graphics in the Cardioid file use images with up to 361 vertices (note that 361 = 19^2)).

The *Sieve* works by systematically eliminating composite numbers using the composite pattens highlighted in each panel.

The first panel highlights all numbers which are multiples of 2. These numbers are removed (except for 2) in Panel 2 which highlights all multiples of the next number, 3. These are removed in Panel 3 and all multiples of next remaining number, 5, are highlighted. Each time we remove highlighted numbers (except for the first) we then highlight multiples of the next remaining number. This number is the next prime. The tables to left and right show the results based on the first 8 primes.

Composites by Panel based on Prime to Delete			Remaining in *Sieve*	
			37 rows of 10	370
Panel	Prime	Delete	less number 1	369
1	2	184	even	185
2	3	61	divisible by 3	124
3	5	24	divisible by 5	100
4	7	13	divisible by 7	87
5	11	7	divisible by 11	80
6	13	4	divisible by 13	76
7	17	2	divisible by 17	74
8	19	1	divisible by 19	**73**

The Panel 1-8 patterns could be extended to find all primes to 529 = 23^2 since 23 is the 9th prime.

The general pattern in each panel is shown in yellow. The first number to be removed is highlighted in **gold**. This number is the square of the prime associated with that panel (since all highlighted composites less than this are multiples of this prime and an (already removed) smaller number).

How can this be extended? Suppose you want to know whether 797 is composite or prime. The square root of 797 is 28.23… . Since this is less than 29, you need only examine the first 9 primes (2, 3, 5, 7, 11, 13, 17, 19, 23). It turns out that it is, indeed, prime (but 799 is composite since 799 = 17·47).