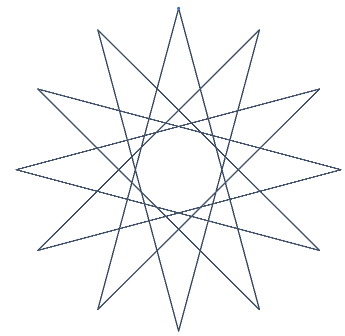
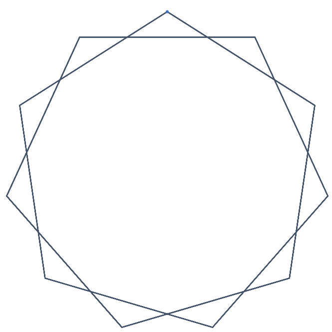
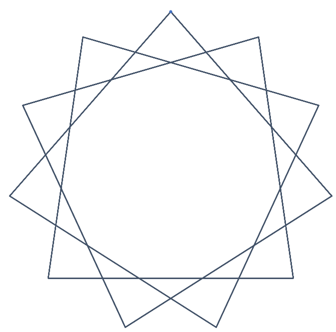
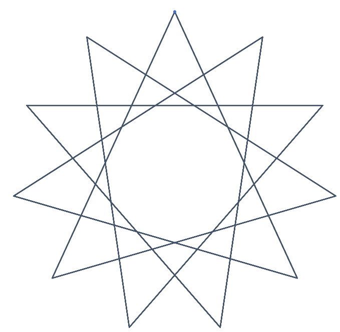
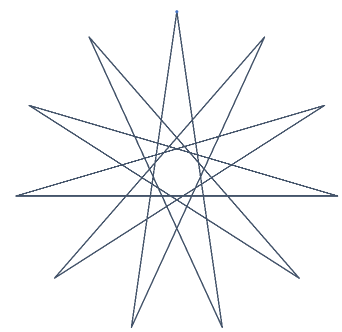
Commonality between Numbers

Prime numbers are the building blocks for both integers and rational numbers. The images created in **PwP** are based on composite numbers as well as prime numbers. For some of the models, when composite numbers are used, the images will “collapse” or look less complete for certain values of other parameters. This happens when there is commonality between the numbers.



Examples of commonality. If the number of vertices in the polygon, ***n***, is even, then when the number of jumps between vertices ***J*** = ***n***/2, the image reduces to a vertical line. Similarly, one can draw a single distinct continuously drawn 12-point star, ***n*** = 12, ***J*** = 5 shown to the right (noting that ***J*** = 7 produces the same image seemingly drawn in a counterclockwise fashion). By contrast, there are four distinct continuously drawn stars when ***n*** = 11 (***J*** = 2, 3, 4, and 5) shown below. The difference is that 2, 3, and 4 are all factors of 12 so that 1/2, $1/3^{rd}$, or $1/4^{th}$ of the vertices is used and a hexagon (6 = 1/2·12), square (4 = 1/3·12), or triangle (3 = 1/4·12) results if ***J*** = 2, 3, or 4, respectively.

Commonality between numbers occurs if the numbers have factors in common. Two numbers may have a number of factors in common with one another. Consider the two numbers ***a*** = 24 and ***b*** = 40.

Factors of ***a***: 1, 2, 3, 4, 6, 8, 12, 24 Factors of ***b***: 1, 2, 4, 5, 8, 10, 20, 40

Factors in common between ***a*** and ***b***: 1, 2, 4, and 8.

The largest of these common factors is called the *greatest common divisor* of ***a*** and ***b***, GCD(***a***, ***b***), so GCD(24, 40) = 8.

Finding the GCD of two numbers. If ***g*** = GCD(***a***, ***b***), then it is the largest number that properly divides both ***a*** and ***b***. By properly divides, we simply mean leaves a remainder of zero upon division by ***g***. This number is simple to find in Excel, by using the GCD function. If you type =GCD(24,40) in a cell and hit Enter, the number 8 is shown in the cell.

It is also easy to find the GCD if you have *complete factorization* of both ***a*** and ***b*** (like the example above). This process is made easier by systematically organizing your factors from smallest to largest (as done above) and compare lists until you find the largest value common to both lists.

You can also find the GCD by comparing *prime factorizations* of both numbers. Using the above, ***a*** = $2^3 \cdot 3$ and ***b*** = $2^3 \cdot 5$ so that GCD(***a***, ***b***) = 2^3. (Prime factorization also allows us to easily identify the *least common multiple* of two numbers which in this case is simply 120 = $2^3 \cdot 3 \cdot 5$.)

Interestingly, factoring large numbers is not something that is easy to do. This fact, discussed in the wonderful book *On Code* by Sarah Flannery, is at the core *public key cryptography*. Of course, the numbers involved in creating public key systems are very large compared to what we work with in **PwP** (imagine trying to factor a 100-digit number).

Despite the difficulty of factoring numbers in general, there is a method to find the GCD of two numbers that does not require knowing the factors of either number. This method, known since ancient times, is called the *Euclidian algorithm* or *Euclid’s algorithm*. Although we do not employ Euclid’s algorithm directly in **PwP**, it is a very elegant algorithm. It can also be used to find *modular multiplicative inverses*, a concept used in understanding some of the more advanced images in **PwP** (don’t worry if you have never heard of modular multiplicative inverses because, as always, one need not know about a topic to enjoy the resulting images). As such, Euclid’s algorithm deserves its own *explainer*.