About Prime and Composite Numbers

Most of the time we restrict our discussion to the natural numbers, 1, 2, 3, … and 0 but there are times when it is useful to consider negative numbers as well. Some authors include 0 among the natural numbers but that is not universally agreed upon and the number 0 has its own very interesting history (as you can see by Googling *who invented zero*).

The natural numbers larger than 1 can be categorized into two types of numbers, those that have proper divisors and those that do not. The term "proper' is included because any natural number, ***n***, can be written as ***n*** = 1·***n*** but 1 and ***n*** are too trivial to count as proper divisors of ***n***.

Definitions: *Proper divisors*. A natural number has proper divisors if it can be expressed as the product of two natural numbers smaller than that number. Divisors are also called *factors*.

For example, 2 and 3 are proper divisors of 6, but 1 and 6 are not.

Composite number. A number is called a *composite* number if it has proper divisors.

The first 10 composite numbers are: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18.

Prime number. If a natural number larger than 1 has no proper divisors, it is called a *prime* number.

The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

One could have defined these terms using multiplication rather than division. A composite number is a number that is a multiple greater than one of another number greater than one. A prime number is a number that is not a multiple greater than one of a number smaller than that number.

Testing for primes. How can you tell if a number is composite or prime? There are some easy rules for looking for factors, but these rules only cover certain factors. Here are 3 simple rules:

1. If a number ends in 0, 2, 4, 6, or 8 it is divisible by 2 and is therefore composite. Any even number larger than 2 is composite.
2. If the sum of the digits is divisible by 3 then the number is divisible by 3.
3. If the number ends in 0 or 5 it is divisible by 5.

Interestingly, using just these three rules, you can find whether any number less than 49 is a prime.

Consider 4 examples: ***n*** = 45. Composite due to rules 2 and 3. Factors are 1, 3, 5, 9, 15, and 45.
n = 46. Composite due to rule 1. Factors are 1, 2, 23, and 46.
n = 47. Prime. Factors are 1, and 47.
n = 48. Composite due to rules 1 and 2. Factors are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

The reason these rules work for numbers up to 49 is that $49 = 7^2$ and 7 is the 4^{th} prime.

If you follow the above three rules and additionally check whether ***n*** is divisible by 7, you can tell whether ***n*** is prime as long as ***n*** < 121. The reason is simple, $121 = 11^2$ and 11 is the 5^{th} prime.

More generally, to test whether ***n*** is prime, you need only test whether the number is divisible by the prime factors less than the square root of ***n*** (in Excel, type =n^0.5 for the square root of ***n***). This idea of checking primes less than the square root of the number you are interested in is central to a method of finding primes that is more than 2000 years old. This method is discussed in the Sieve of Eratosthenes *explainer*, and it works by using the patterns created by multiples of primes to remove composites from an array of numbers.

More on factors. It is worth noting that if divisors are listed from smallest to largest then they are connected to one another in a very specific fashion. The first and last multiply to ***n***; the second and second-to-last multiply to ***n***, and so on. (So, given the factors of ***n*** = 48 listed above: 1·48 = 48; 2·24 = 48; 3·16 = 48; 4·12 = 48; and 6·8 = 48.)

This pairing up of smaller factors with larger ones means that, in general, a number will have an even number of factors. This is true unless ***n*** is a perfect square. In this case, the middle factor squared is the number. For example, the perfect square 16 has five factors listed from smallest to largest: 1, 2, 4, 8, 16. The middle factor is 4 and $4^2 = 16$.