

There are only Two Ways to Draw an Image

Suppose you stumble onto an image you like. [Explainer 2.4a](#) argues that a reasonable strategy to understand how the image is made is to look for the smallest numbers for n , S , P , and J that produce the image. This can be accomplished by simplifying the vertex frame and by reducing the size of P and S .

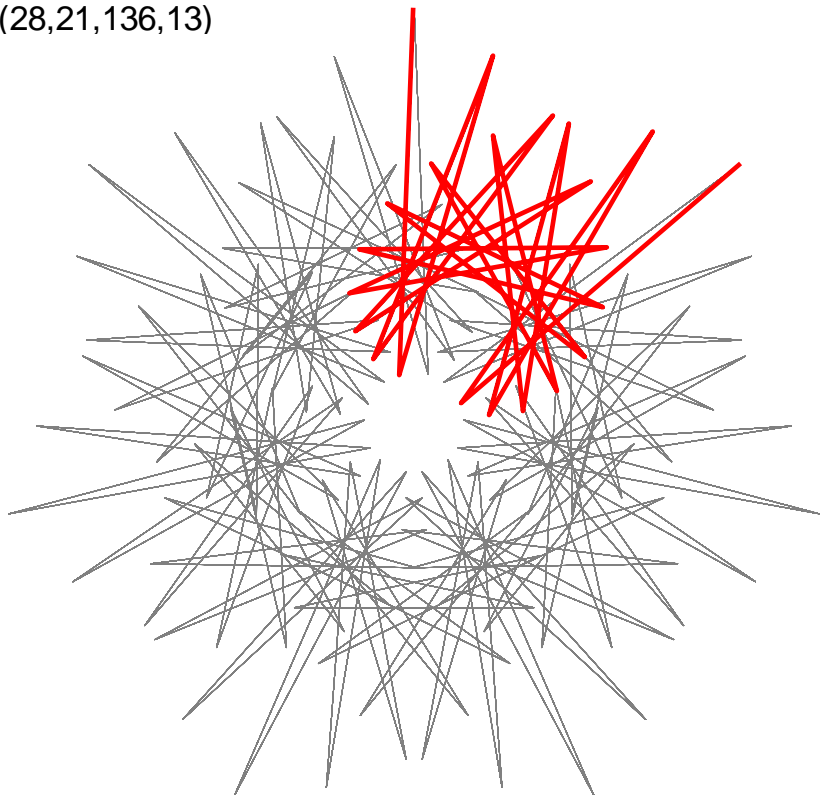
Simplifying the vertex frame. First, we restrict ourselves to $J < n$ because $J > n$ simply means counting more than once around the polygon before finding the next vertex to jump to on the vertex frame. Second, we remove common factors from J and n so that $VCF = GCD(J, n) = 1$. And finally, by choosing $J < n/2$, the vertex frame is drawn in a clockwise fashion.

Reducing the size of P and S . First, note that if $P > S \cdot n$ you can get the same image by subtracting as many multiples of $S \cdot n$ as you need to from P in order to have $P < S \cdot n$. The reason is simple: There are $S \cdot n$ subdivisions total so why count more times around than necessary before finding the endpoint for the next line in the image? Second, the simplest way to reduce S and P is to make sure they have no common factor among them. If there is a common factor, then divide both by that factor until $GCD(S, P) = 1$. Additionally, Finally, note that $P = P_0$ and $P = S \cdot n - P_0$ produce the same image. As a result, we can choose the one that is smaller than $S \cdot n/2$.

The strategy outlined above produces the values listed for the *Spinning Needle Star* shown in the top left corner of the **Playing with Polygons** home page (and referred to in the first paragraph of the home page noting the [link to the companion website](#)).

(n, S, P, J)
(28,21,136,13)

147 lines



Here is that image with first cycle shown in red. With the cycle information shown it is quite clear, even without clicking *Toggle Drawing*, that this spinning star is created in 7 clockwise cycles around the polygon. Every fourth vertex is used, but the other vertices help create the spiky image.

Suppose we were contrarians who wanted to have the SAME image spin in a counter-clockwise direction. We could accomplish this in two ways.

Replace J with $n-J$: <https://www.playingwithpolygons.com?vertex=28&subdivisions=21&points=136&jumps=15> .

Replace P with $S \cdot n - P$: <https://www.playingwithpolygons.com?vertex=28&subdivisions=21&points=452&jumps=13> .

The question naturally arises: What if you do both? In that instance, the image is once again clockwise, just like above: <https://www.playingwithpolygons.com?vertex=28&subdivisions=21&points=452&jumps=15> .

More generally, if $J_0 < n_0$, $P_0 < n_0 \cdot S_0$ then (n_0, S_0, P_0, J_0) , $(n_0, S_0, P_0, n_0 - J_0)$, $(n_0, S_0, n_0 \cdot S_0 - P_0, J_0)$ and $(n_0, S_0, n_0 \cdot S_0 - P_0, n_0 - J_0)$ all produce the same image. The middle two are drawn one way and the outer two are drawn the other.